§40. Diamagnetic Measurements and Plasma Energy in Toroidal Systems

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Sensitivity of the diamagnetic signal to several operational and geometrical factors is analysed. Among them are the flux conservation in the plasma, eddy currents induced in the outer structures at fast processes, toroidal shift and deformation of the plasma boundary due to its energy change, and inhomogeneity of the confining magnetic field. It is shown that in each case, under proper experimental circumstances, the contribution, unaccounted in the traditional theory of diamagnetic measurements, can reach a level compared to β (ratio of the volume-averaged plasma pressure to the magnetic field pressure). The approach is fully analytical with all relevant dependencies shown explicitly, allowing easy estimates and suggesting a resolution of the problem in order to restore the accuracy of finding β from diamagnetic measurements. This essentially extends the analysis [1] of possible measures to improve separation of the useful fraction of the measured diamagnetic signal. The approach is aimed to explanation of the discrepancies between model estimates and experimental results, unification of a knowledge obtained in separate numerical studies, extending a theoretical basis of magnetic diagnostics and uncovering potential dangers in interpretations. This is also an essential step from traditional cylindrical theory to analytical derivations in the toroidal geometry. The results are equally applicable to tokamaks and stellarators.

Interpretation of diamagnetic measurements in tokamaks and stellarators is based on a simple formula

$$2\frac{\Delta\Phi}{\Phi_{pl}} = \frac{B_J^2}{B_0^2} - \beta , \qquad (1)$$

derived for a circular plasma cylinder more than 50 years ago, or its modifications. Here

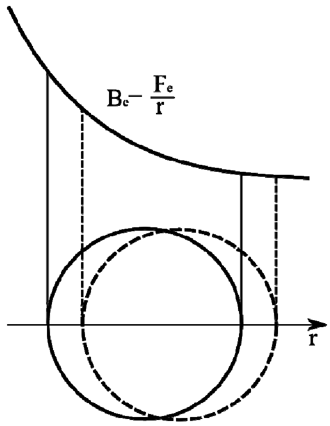
$$\Delta\Phi = \int_{loop} (\mathbf{B} - \mathbf{B}_v) \cdot d\mathbf{S}_l , \qquad (2)$$

Φ is the flux of the magnetic field through the diamagnetic loop and $\Delta\Phi$ is the difference between the current state and initial state when $B_J = \beta = 0$, $\mathbf{B}$ is the magnetic field, $\mathbf{B}_v$ is the vacuum magnetic field (assumed unchanged in this case), $\Phi_{pl} = B_0 S_{pl}$ with $S_{pl} = \pi b^2$ the transverse cross-section of the plasma column, b is its minor radius, B_0 is the toroidal field, B_J is the poloidal field at the plasma boundary (the field of the net toroidal current), β is the ratio of the volume-averaged plasma pressure p to the magnetic field pressure $B_0^2/2$.

Sometimes (at fast heating, for example) the plasma evolution can be considered as flux-conserving. Then the diamagnetic loop can measure only the variation of the magnetic flux between the plasma and the loop. The result depends on the boundary conditions for the magnetic field in the plasma-wall vacuum gap. An important contribution comes from the plasma expansion with β rise. This problem is analyzed in detail and proper expressions for several cases of interest are presented in [2].

It is also shown that that disregard of the Shafranov shift in the diamagnetic measurements results in slightly underestimated β. More important can be dependence of the diamagnetic signal on the plasma global shift.

The latter effect is strictly toroidal, which is not included in (1) or its modifications within cylindrical models. It is related to $1/r$ dependence of the toroidal field, so that the outward shift brings the plasma to the region of weaker magnetic field, as shown in the Figure: Two positions of the plasma and the external toroidal field $B_e = F_e / r$. The shift can appear when β increases and can be large enough [3] in LHD experimental conditions. If the plasma evolution is flux-conserving, the diamagnetic loop will measure the signal [2]



$$\delta\Phi_{gap} = S_{gap}\delta B_e - B_e \delta S_{pl} + \Phi_{pl}\frac{\delta\Delta_b}{R} , \qquad (3)$$

where δ means the increment, S_{gap} is the surface of the gap between the plasma and the diamagnetic loop, S_{pl} is the surface of the plasma cross-section, Φ_{pl} is the total toroidal flux in the plasma, Δ_b is the plasma shift and R is the major radius. It is important that flux conservation in the plasma results in nonzero δS_{pl}.

Estimates show that the last term in (3), not included into the cylindrical model, can give an unaccounted contribution to the measured diamagnetic signal comparable to the signal itself. Note that the geometrical effects related to the plasma boundary change and the toroidal effects do not depend on a particular model of the plasma. Therefore, we can easily combine the "geometrical" part of the presented theory with any alternative approach to the plasma equilibrium.

With proper definitions, equation (3) can be used when the plasma volume is changed as found in [4].

[1] T. Yamaguchi et al., *Plasma Phys. Control. Fusion,* **48**, L73 (2006).
[2] V.D. Pustovitov, *Plasma Phys. Control. Fusion*, **52**, 085005 (2010).
[3] V.D. Pustovitov, *Reviews of Plasma Physics*, ed. B.B. Kadomtsev and V.D. Shafranov (New York: Consultants Bureau), 2000, Vol. 21, pp. 1–201.
[4] K.Y. Watanabe et al., *Plasma Phys. Control. Fusion*, **49**, 605 (2007).