§13. Development of Next-Generation High-Resolution Scheme for Magnetohydrodynamics

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A next-generation very high-resolution numerical scheme for magnetohydrodynamics (MHD) in the petafrop era is developed in this study. In the next-generation MHD simulations, realistic MHD simulations with extremely wide range of scale should be accomplished, and therefore, it will be expected that the picture of macroscopic structure and dynamics of plasmas is dramatically changed. However, since current numerical methods for MHD are insufficient for numerical stabilities in the high-resolution simulation, those cannot accurately and robustly simulate multi-scale structure and dynamics of MHD. Also, those are not computationally efficient enough in order to go to the next level. Thus, the development of a robust and efficient high-resolution MHD scheme is one of the most important tasks in high performance computing for MHD. Particularly, in this paper, a shock capturing scheme is paid attention to. As a basic scheme, we adopt an approximate Riemann solver such as the Harten-Lax-van Leer-Discontinuities (HLLD) Riemann solver, which is an accurate, robust, and efficient solver for MHD. [1]

Though the approximate Riemann solver is quite robust in one-dimension, a multi-dimensional extension of that is not straightforward because, in general, the approximate Riemann solver is based on the one-dimensional theory of characteristics. Particularly for MHD, unphysical divergence of the magnetic field is numerically produced in multi-dimensions, and therefore, the multi-dimensional simulation can be abnormally terminated due to unphysical Lorentz force. Therefore, several divergence cleaning techniques including a new HLL-type Constrained Transport (CT) method are comparatively studied. The CT method is a powerful candidate for our next-generation scheme because it is completely divergence-free using the electric field on the cell edges. However, since the electric field on the edges is evaluated by a simple arithmetic mean of cell-faced numerical fluxes in the standard flux-CT method, the multi-dimensional scheme is not consistent with the base one-dimensional scheme when a one-dimensional data is computed.[2] It is not assured that the robustness of the base scheme is taken over in the standard flux-CT method. Therefore, we construct an HLL-type CT method where the electric field on the edges is evaluated considering the gradient of the electric field. Here, the gradient of the electric field is computed using the HLL flux of the induction equation. It is found that the HLL-type CT method is not only consistent with the base scheme in one-dimension but also robust enough in multi-dimensions.

It is well known that more than second-order *linear* schemes are numerically unstable. Modern higher-order schemes like Monotone Upstream-centered Scheme of Conservation Laws (MUSCL) [3] and Piecewise Parabolic Method (PPM) [4] that are *nonlinear* schemes can be stabilized by introducing limiter functions. To construct these higher-order schemes for the linear scalar advection equation, the distribution of the scalar variable in the cell is reconstructed using the gradient of the variable controlled by the limiter function. The scalar variable corresponds to the conservative variable as well as the characteristic variable. On the other hand, for nonlinear systems of hyperbolic conservation laws such as the Euler equations and the MHD equations, the reconstruction of the distribution of the variables can be carried out by using some variables as the conservative variables, the primitive variables, and the characteristic variables. In general, using the characteristic variables, numerical oscillations can be suppressed. However, particularly for MHD, numerical calculations of the characteristic variables are extremely time-consuming. Therefore, we newly introduce a set of approximate characteristic variables that are given by splitting the MHD equations into compressible and incompressible parts. It is confirmed from some numerical tests that the reconstruction through the approximate characteristic variables are not only more efficient than that using the characteristic variables but also more stable than that using the primitive variables.

As a physical problem, using our next-generation MHD scheme, we perform a very high-resolution simulation of the nonlinear resistive tearing instability at high Lundquist number. Simulation results show that multiple small scale plasmoids are intermittently created by the secondary tearing instabilities at the nonlinear stage and are ejected faster than the local fast magnetosonic speed (Fig. 1). We find that the reconnection rate is greatly enhanced with intermittent plasmoid ejections. These results imply that the nonlinearity of MHD at very high Lundquist number may be effective for explosive magnetic reconnection through the self-destabilization of micro-scale instabilities.

Papers for details of the numerical techniques and the nonlinear resistive tearing instability at high Lundquist number are in preparation.

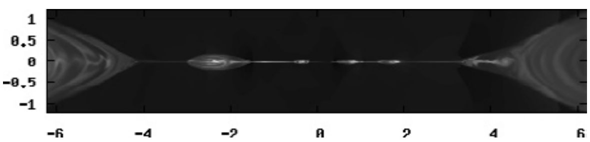

Fig. 1. Gray scale image of the temperature of the nonlinear resistive tearing instability at high Lundquist number.

1) Miyoshi, T., Kusano, K.: J. Comput. Phys. **208** (2005) 315
2) Gardiner, T. A., Stone, J. M.: J. Comput. Phys. **205** (2005) 509
3) van Leer, B.: J. Comput. Phys. **23** (1977) 276
4) Collela, P., Woodward, P. R.: J. Comput. Phys. 54 (1984) 174