§17. Physical Space Energy Cascade in Developed Turbulence

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It is believed that small-scale statistics of turbulent flows are universal irrespective of the external forcing and/or the boundary conditions of each flow. For example, Kolmogorov's $-5/3$ power law of the energy spectrum is always observed in any turbulent flow at high Reynolds numbers, and the coefficient of the power law seems to be universal (Fig. 1). This small-scale universality is explained by the *energy cascade*: that is, the process that energy supplied to the system by an external force (or from its boundary) at large scales transfers to small scales. If this energy cascade takes place scale by scale, the information of the force or the boundary (which is non-universal) might well be forgotten through the process. Therefore, the energy cascade is a key notion to understand the universality or non-universality of turbulent flows.

Indeed, the energy transfer from lower to higher wavenumbers in the Fourier space is evident numerically and theoretically. However, its physical mechanism is not well understood. We suggested [1)], based on relatively small numerical simulations of homogeneous isotropic turbulence, that the energy cascade could be caused by the vortex stretching of smaller-scale (i.e. thinner) vortex tubes in the straining field created by larger-scale (i.e. fatter) vortex tubes. Especially, anti-parallel pairs of vortex tubes are likely to play an essential role to create strong straining fields. However, in our previous study, we could not simulate turbulence enough developed to verify the proposed scenario of the energy cascade. Therefore, we need to investigate turbulence at higher Reynolds numbers.

On the Plasma Simulator, we have carried out direct numerical simulation[2)] of an incompressible fluid in a periodic cube with 2048^3 grid points. The Taylor-length based Reynolds number of thus simulated turbulence is about 540. The energy spectrum of the field is plotted in Fig. 1 by the solid curve together with some experimental results. A clear inertial range (where the spectrum obeys Kolmogorov's $-5/3$ power law) is observed in our simulation. In other words, we are now ready to numerically analyse the inertial range features such as the energy cascade process.

It is quite easy to find events in the developed turbulence that support the energy cascade scenario proposed in Ref. 1). An example is shown in Fig. 2. In this figure, an anti-parallel pair of vortex tubes at a large scale create thinner vortex tubes. Note that created thinner vortex tubes align to each other in the perpendicular direction to the fatter vortex pair. This observation is consistent with the scenario, since the straining due to the larger (fatter) vortex tubes is strongest in the direction. Furthermore, we are developing chasing algorithm of individual vortex tube at each scale to investigate its life. More quantitative study, by the use of this algorithm, to support the proposed scenario of the energy cascade is under investigation.

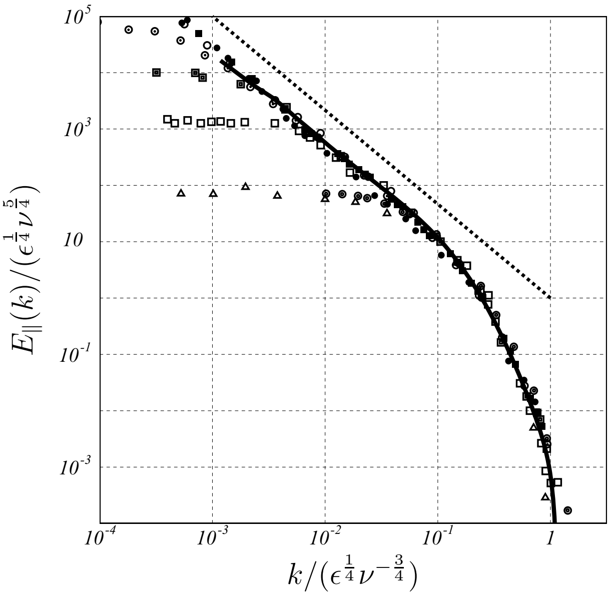

Fig. 1: Energy spectrum. Solid curve, the result of our numerical simulation; symbols, experiments and measurements of different flows. The dotted line indicates Kolmogorov's $-5/3$ power law.

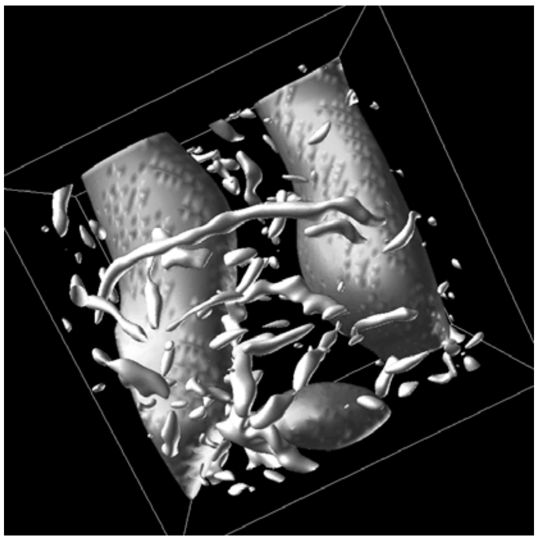

Fig. 2: An example of the creation of smaller-scale vortex tubes by stretching in a straining field around an anti-parallel pair of larger-scale vortex tubes. It can be observed that created thin vortex tubes tend to align to each other in the perpendicular direction to the parent vortices.

1) S. Goto: J. Fluid Mech. **605** (2008) 355.
2) S. Goto: Advances in Turbulence **12** (2009) 269.