§19. Study of Field Singularities and Inter-scale Energy Transfer in NS and MHD Turbulence by Using Massive Parallel Direct Numerical Simulation

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One of the most unique and important features of NS and MHD turbulence is the fact that the statistical distribution of the fluctuations changes with decrease of spatial scale. But strong nonlinearity and non-equilibrium state prevents from full understanding of the turbulence physics. Singular behavior of the field and the energy transfer from the large energy containing range to the dissipation range due to the inertial action of the fluid are related internally through the inviscid constants of the fundamental equation which depend on the spatial dimension d. It is well known that the inviscid constants are different in even and odd dimensions, and that only the constants at the second order in amplitudes are actually conserved in the direct numerical simulation (DNS) of turbulence, for example, the energy and enstrophy in two dimensions and the energy and helicity in three dimensions.

I this study, we have performed DNSs of the incompressible isotropic turbulence in space of dimension five. The initially random velocity field decays in time and the fundamental statistics such as the energy spectrum, the total energy, enstrophy are gathered. The probability density functions of the eigenvalues of the strain tensor are also analyzed. The normalization and Taylor's microscale Reynolds number are defined by

$$E = \frac{d}{2}u'^2 = \int_0^\infty E(k)dk, \quad R_\lambda = \frac{u'\lambda}{\nu}.$$

Note that the average kinetic energy in each direction is initially equal.

The Reynolds number is too low for the Kolmogorov spectrum $E_d(k) = K_d \bar{\epsilon}_d^{2/3} k^{-5/3}$ to be observed, but it was found that the spectrum in five dimensions normalized by the Kolmogorov variables shifts toward higher wavenumbers and becomes smaller in amplitudes when compared to those in three and four dimensions (see Figure 1). This observation is consistent with the theoretical prediction of the Kolmogorov constant ($K_3 = 1.72, K_4 = 1.31, K_5 = 1.16$) [1,2].

Figure 2 shows the comparison of the normalized PDFs of the eigenvalues in dimensions three, four, and five. The abscissa is normalized by the average rate of the energy dissipation per unit mass $\sqrt{\bar{\epsilon}_d/\nu}$. It can be seen that when the dimension increases the PDFs of the largest and smallest eigenvalues change little, while the PDFs of the intermediate eigenvalues tend to equally be distributed in between the largest and smallest eigenvalues. The PDFs of the largest and smallest eigenvalues in dimension three and five are closer to each other, and the left tail of the PDF of the smallest eigenvalue in dimension five is longer than those of PDFs in three and four dimensions. This is consistent with the theoretical predition that when the spatial dimension increases the incompressible condition becomes so less restrictive that the longitudinal negative velocity gradient (compression) can be larger [2].

[1] E. Suzuki, T. Nakano, N. Takahashi, and T. Gotoh, Phys. Fluids **17**, 081702 (2005).

[2] T. Gotoh, Y. Watanabe, Y. Shiga, T. Nakano, and E. Suzuki, Phys. Rev. E**75**, 016310 (2007).

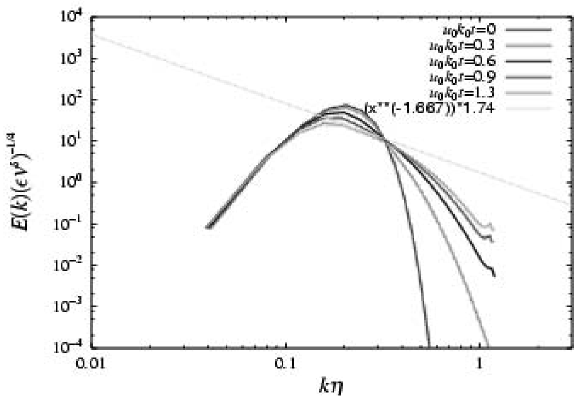

FIG. 1: Evolution of the scaled energy spectrum of five dimensional decaying turbulence. Straight line shows the Kolmogorov spectrum with the constant $K_5 = 1.16$ in 5D .

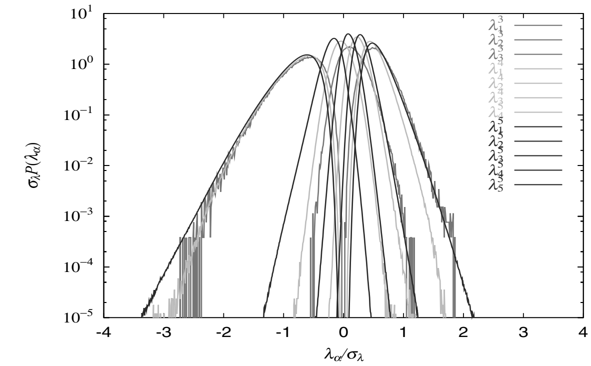

FIG. 2: Comparison of the PDFs of the eigenvalues of the strain tensor in 3D, 4D and 5D. $R_\lambda = 14$ for three cases.