# §3. Analytic High-Beta Tokamak Equilibria with Poloidal-Sonic Flow and Pressure Anisotropy

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Analytic high-beta tokamak equilibria with flow comparable to the poloidal sound velocity[1)] have been extended to include pressure anisotropy. Pressure anisotropy is relevant for plasma flows driven by neutral beam injection. A reduced set of equilibrium equations has been derived from single fluid magnetohydrodynamic (MHD) equations and equations for the parallel and perpendicular pressures and the parallel heat fluxes for ions and mass-less electrons as an extension from the isotropic and adiabatic pressure case[2)]. We have adapted a fluid closure model[3)] where fourth-order moments are approximated to products of second-order ones. Asymptotic expansions in terms of the inverse aspect ratio $\varepsilon$ of high-beta tokamaks, $p\sim\varepsilon B_0^2/\mu_0$ and $B_p\sim\varepsilon B_0$, have been applied. We assume strong pressure anisotropy, $p_{||}-p_{\perp}\sim p$. The resulting equations are Grad-Shafranov type equations for the first- and second- order magnetic flux. These can be solved analytically when the pressures and the square of the Alfven Mach number are linear with the first-order magnetic flux,

$$p_{\{i,e\}\{\|,\perp\}1} = \varepsilon\left(B_0^2/\mu_0\right)p_{\{i,e\}\{\|,\perp\}1c}\bar{\psi}_1, \quad M_{Ap}^2 = \varepsilon M_{Apc}^2\bar{\psi}_1. \qquad (1)$$

Figures 1 show profiles of an analytical solution; (a) the magnetic structure is modified by the flow due to the centrifugal force and through the Bernoulli law, (b) the pressure isosurfaces depart from magnetic flux surfaces due to the poloidal flow and (c) anisotropic pressure profiles are self-consistently determined in the presence of flow. Figure 2 shows the shift of the magnetic axis from the geometric axis as a function of the square of the poloidal Mach number. There are three singular points where the poloidal flow velocity equals the phase velocities of either slow magnetosonic or two ion acoustic waves,

$$M_{Apc}^2 = \frac{1}{2}\; 6p_{i\|1c} + p_{e\|1c} \pm \sqrt{24p_{i\|1c}^2 + p_{e\|1c}^2}\;, \quad p_{i\|1c}, \qquad (2)$$

which arise from heat flux equations[3)]. This result indicates a qualitative difference from the previous one obtained with adiabatic pressure[1)], where the only one singular point corresponding to the slow magnetosonic wave exists.

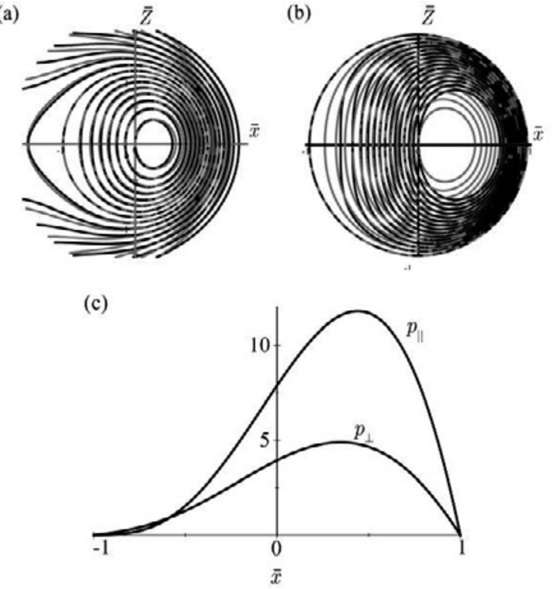


Fig.1 Analytical solution for single-fluid equilibrium with flow: (a) the magnetic flux surfaces (black) compared with its static case (gray), (b) the isosurfaces of the average pressure $(p_{||}+p_{\perp})/2$ (black) and the magnetic flux surfaces (gray) and (c) radial profiles of $p_{||}$ and $p_{\perp}$ in the midplane.

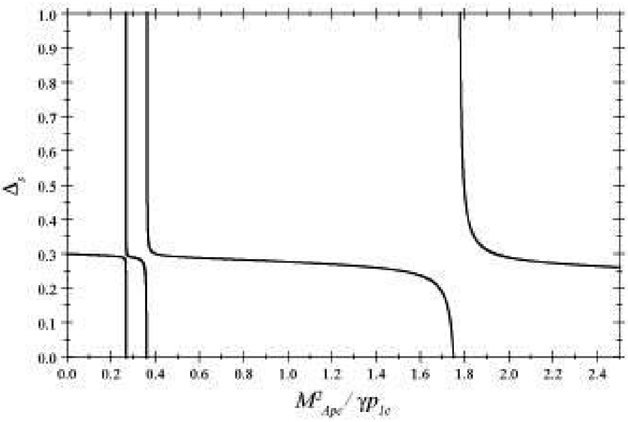


Fig.2 The shift of the magnetic axis from the geometric axis as a function of the square of the poloidal Mach number.

1) Ito, A. and Nakajima, N.: Plasma Phys. Control. Fusion **51**, 035007 (2009) [*Corrigendum*, Plasma Phys. Control. Fusion **52**, 079802 (2010)].
2) Ito, A., Ramos, J. J., and Nakajima, N.: Plasma Fusion Res. **3**, 034 (2008).
3) Ramos, J. J.: Phys. Plasmas **9**, 3607 (2003).