§4. Effects of Finite Larmor Radius and Pressure Anisotropy on Equilibria with Flow in Reduced Two-Fluid Models

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Flows in magnetically confined plasmas may play an important role for the formation of steep structure where the scale lengths of microscopic effects cannot be neglected. We investigate effects of flow, ion finite Larmor radius (FLR) and pressure anisotropy on macroscopic equilibrium of a high-beta toroidal plasma based on reduced magneto-hydrodynamic (MHD) models. A novel set of reduced equilibrium equations for high-beta tokamaks with toroidal and poloidal flow in the order of poloidal sound velocity has been formulated from two-fluid MHD equations with ion finite Larmor radius (FLR) terms and pressure anisotropy.

The reduced set of equilibrium equations is derived from two-fluid MHD equations by means of asymptotic expansions in terms of the inverse aspect ratio ε and the ratio δ of the ion Larmor radius to the minor radius for high-beta tokamaks, $p\sim\varepsilon B_0^2/\mu_0$ and $B_p\sim\varepsilon B_0$. With the drift ordering where the flow velocity is order δ of the ion thermal velocity, the ion FLR terms for collisionless plasmas like the gyroviscosity and the diamagnetic heat flux are much simplified. Poloidal-sonic flow is introduced by setting $\varepsilon\sim\delta$. In order to include flow and the non-ideal effects under these orderings, a higher-order accuracy compared to the standard reduced MHD equations is required[1,2]. Pressure anisotropy is relevant for plasma flows driven by neutral beam injection and is introduced with the equations for the ion and electron parallel heat fluxes incorporating a fluid closure model where fourth-order moments are approximated with products of second order ones. The set consists of the first two orders of the Grad-Shafranov equation of which the first order is same as that for static equilibria,

$$\left(\frac{\partial^2}{\partial R^2}+\frac{\partial^2}{\partial Z^2}\right)\psi_1 = -\mu_0 R_0^2\left[\left(\frac{x}{R_0}\right)\sum_{s=i,e}(p'_{s\|1}+p'_{s\perp 1})+g'_*\right]-\left(\frac{I_1^2}{2}\right)', \tag{1}$$

and the second order is given by

$$\begin{aligned}&\left(\frac{\partial^2}{\partial R^2}+\frac{\partial^2}{\partial Z^2}\right)\psi_2 \\ &+\left\{\mu_0 R_0^2\left[\left(\frac{x}{R_0}\right)\sum_{s=i,e}\left(p''_{s\perp 1}+p''_{s\|1}\right)+g''_*\right]+\left(\frac{I_1^2}{2}\right)''\right\}\psi_2 \\ &-\frac{1}{R}\frac{\partial\psi_1}{\partial R}-F\left(\frac{\partial^2}{\partial R^2}+\frac{\partial^2}{\partial Z^2}\right)\psi_1-F'\frac{|\nabla\psi_1|^2}{2} \\ &+\mu_0 R_0^2\left[E'_*+\left(\frac{x}{R_0}\right)\sum_{s=i,e}\left(P'_{s\perp 2*}+P'_{s\|2*}\right)\right. \\ &\left.+\frac{1}{2}\left(\frac{x}{R_0}\right)^2\sum_{s=i,e}\left(p'_{s\perp 1}+p'_{s\|1}+C'_{s\perp}+C'_{s\|}\right)\right]=0,\end{aligned} \tag{2}$$

$$\begin{aligned}F(\psi_1)=&\left[m_i n_0 R_0^2\left(\Phi'_1+\frac{\lambda_H-\lambda_i}{en_0}p'_{i\perp 1}\right)\left(\Phi'_1+\frac{\lambda_H}{en_0}p'_{i\perp 1}\right)\right. \\ &\left.+\sum_{s=i,e}(p_{s\|1}-p_{s\perp 1})\right]\left(\frac{B_0^2}{\mu_0}\right)^{-1},\end{aligned} \tag{3}$$

where λ_H and λ_i label effects of two fluids and the ion FLR respectively: $(\lambda_H, \lambda_i)=(0, 0)$ for single fluid (ideal) MHD, $(\lambda_H, \lambda_i)=(1, 0)$ for two-fluid MHD with zero ion Larmor radius and $(\lambda_H, \lambda_i)=(1, 1)$ for two-fluid MHD with ion FLR. Equation (3) includes terms representing the $E\times B$ and the ion diamagnetic flows with the gyroviscous cancellation and pressure anisotropy. While $C_{s\|,\perp}(\psi_1)$ are obtained by solving the equations for the second-order quantities, other coefficients are arbitrary functions of ψ_1. The pressure and the ion stream function are self-consistently determined and their second-order quantities,

$$p_{s\{\|,\perp\}2}=p'_{s\{\|,\perp\}1}\psi_2+\left(\frac{x}{R_0}\right)C_{s\{\|,\perp\}}(\psi_1)+P_{s\{\|,\perp\}2*}(\psi_1), \tag{4}$$

$$\Psi_2=\Psi'_1\psi_2+\lambda_H\left(\frac{x}{R_0}\right)C_{\Psi *}(\psi_1)+\Psi_{2*}(\psi_1), \tag{5}$$

indecate the deviation from magnetic flux functions, which are key characteristics of flowing two-fluid equilibria. A numerical code for solving Eq. (2) with a finite element method is being developed.

1) Ito, A. and Nakajima, N.: AIP Conf. Proc. **1069**, 121 (2008).
2) Ito, A., Ramos, J. J., and Nakajima, N.: Plasma Fusion Res. **3**, 034 (2008).