§25. Analysis of Topological Structure Formation in Hall MHD by Orthonormal Vector Wavelets

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A detailed analysis of the forward and inverse energy transfer process due to the Hall term effect in a freely decaying, homogenerous, isotropic Hall magnetohydrodynamics (HMHD) turbulence is carried out in terms of the Fourier and wavelet analysis. We analyzed three snapshot data that are taken from such a period of time that the turbulence is sufficiently developed and its magnetic Reynolds number is almost constant. Since the Fourier energy spectra of these snapshots show remarkable agreement after the normalization in terms of the dissipation rates and the diffusion coefficients, they are considered in a universal equilibrium state.

Normalized Fourier spectra of the magnetic energy transfer for these three snapshots are shown in Fig.1. In order to compare the different time results, all the abscissas and ordinates of Fourier spectra are made dimensionless by using the diffusion coefficient and the dissipation rate of the field, i.e., η and $\epsilon^{(b)}(t) := -\eta \int \boldsymbol{b}(t)\cdot\nabla^2\boldsymbol{b}(t)\mathrm{d}^3\vec{x}$ where η is the magnetic diffusion coefficient. The wave number and the transfer functions are normalized by $k_\eta^{(b)}(t) := \left(\epsilon^{(b)}(t)/\eta^3\right)^{1/4}$ and $T_\eta^{(b)}(t) := (\eta\epsilon^{(b)}(t))^{3/4}$. Due to this normalization, the amplitudes of spectra become comparable for different times, which makes us easy to estimate the relative contribution of each term to the dynamics.

By analyzing the numerical solutions that are generated without any external forcing, it is confirmed that the inverse energy transfer due to the Hall term effect is intrinsic to the HMHD dynamics. However, the contribution of the inverse energy transfer due to the Hall term effect to the whole transfer is rather small.

It is interesting that the normalized magnetic energy transfer function $T_b(t)/T_\eta^{(b)}(t)$ is almost stationary, while the amplitudes of the induction and the Hall term effect gradually reduce. This implies that the forward transfer due to the magnetic induction is remarkably compensated by the inverse transfer due to the Hall term effect so that the magnetic energy is transferred to the smaller scales in a self-similar manner as a whole. So it is conjectured that the inverse energy transfer due to the Hall term effect work as such a kind of regulating process that retains the whole magnetic energy transfer process in a self-similar state.

Orthonormal divergence-free wavelet analysis reveals that the nonlinear mode interactions that contribute to the inverse energy transfer have nonlocal feature, while for the forward transfer is dominated by local interactions.

These results are presented at ITC20.

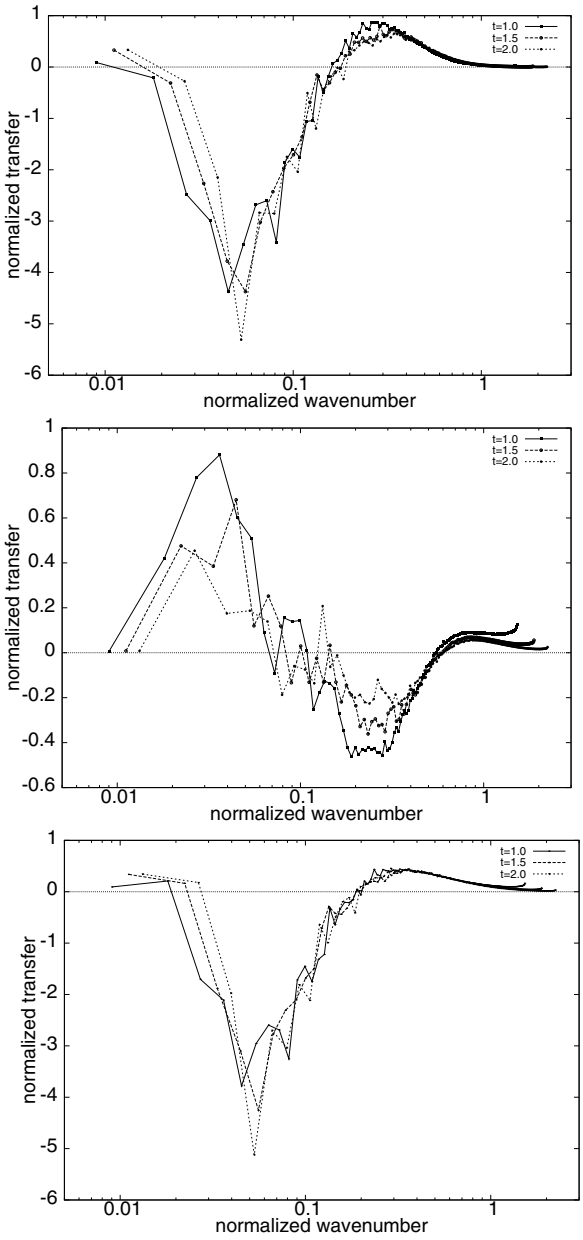

Fig. 1: Normalized Fourier spectra of the transfer functions for the magnetic induction process (top), the Hall term effect (middle), and their sum, i.e., the whole magnetic energy.