§17. Turbulence Sustained in a Precessing Sphere

Goto, S., Shimizu, M. (Osaka Univ.)

The rotation of the spin axis of a rotating object is called the precession. Since the seminal experiment by Malkus in 1968, it is widely known that strong turbulence may be sustained in a weakly precessing cavity. This fact has been emphasized mainly in the geophysics, since the Earth is weakly precessing; but it is also important in engineering applications because a new type of mixer without impellers, for example, may be developed by the use of a precessing vessel.

For a given shape of container (in the following, we consider the spherical one) and for a fixed angle (in our case, it is fixed at a right angle) between the spin and the precession axes, only two non-dimensional parameters, i.e. the Reynolds number, $Re = a^2\Omega_s/\nu$, and the Poincaré number, $\mathit{\Gamma} = \Omega_p/\Omega_s$, control flows inside the precessing cavity. Here, a, Ω_s, Ω_p and ν are the radius of spherical cavity, the magnitude of spin and precession angular velocities and the kinematic viscosity of the confined fluid.

One of the most interesting characteristics of this system is that a wide variety of (steady, periodic, and turbulent) flows are sustained in a precessing cavity depending on the two parameters Re and $\mathit{\Gamma}$. In order to reveal the flow dependence on these parameters, we have conducted laboratory experiments. Our systematic parameter survey on the basis of velocity measurements (the particle image velocimetry) show that strong turbulence is sustained in the precessing sphere when $Re > O(1000)$ and $0.01 \lesssim \mathit{\Gamma} \lesssim 0.1$. It is also experimentally shown that large-scale structures of turbulent flows are determined only by the Poincaré number $\mathit{\Gamma}$, and they are almost independent of the Reynolds number Re.

However, it is rather difficult to investigate the three-dimensional flow structures, the mixing ability and detailed parameter dependence (i.e. critical parameters of transitions) of flows sustained in the precessing sphere only by laboratory measurements. On the other hand, since the boundary condition of this system is so simple that one may conduct direct numerical simulations (DNS) under the precisely same conditions as in the laboratory experiments. So, in the present study, we have conducted, on the Plasma Simulator, DNS of turbulence sustained in the precessing sphere.

Our numerical scheme is as follows. Due to the incompressiblity of the fluid, the velocity field $\boldsymbol{u}(\boldsymbol{x},t)$ is expressed by two scalar functions $U(\boldsymbol{x},t)$ and $W(\boldsymbol{x},t)$ as $\boldsymbol{u} = \boldsymbol{\nabla}\times\boldsymbol{\nabla}\times(U\boldsymbol{x}) + \boldsymbol{\nabla}\times(W\boldsymbol{x})$, and the Navier-Stokes equations lead to the governing equations for U and W. Then, these equations are numerically integrated by the Crank-Nicolson method for the viscous term, and the Adams-Bashforth method for the other terms. The spatial derivatives in their governing equations are evaluated by the spectral method, where U and W are expanded in terms of the spherical harmonic functions and the Zernike spherical polynomials.

A numerical result is shown in Fig. 1(a) together with the corresponding experimental measurement (b). In this figure, the magnitude of turbulence intensity on the central plane perpendicular to the spin axis is plotted in a gray scale (contour levels are common in the two figures); i.e. turbulence is stronger in brighter regions. The two control parameters are set at $Re = 10000$ and $\mathit{\Gamma} = 0.1$ so that we can investigate fully developed turbulence case. It is clearly verified that the DNS reproduces the experimental data accurately. Therefore, we may investigate detailed proprieties of the turbulence by using the DNS data. For example, it is shown that the strongly turbulent regions observed in the left-bottom and right-top parts of this figure consist of a ring-like structure in the three-dimensional space. Furthermore, we are conducting DNS to estimate the mixing ability of the turbulence by tracking fluid particles advected by it. A preliminary result shows that the mixing in the sphere can be completed within about a dozen spin periods, when the Poincaré number $\mathit{\Gamma} = 0.1$.

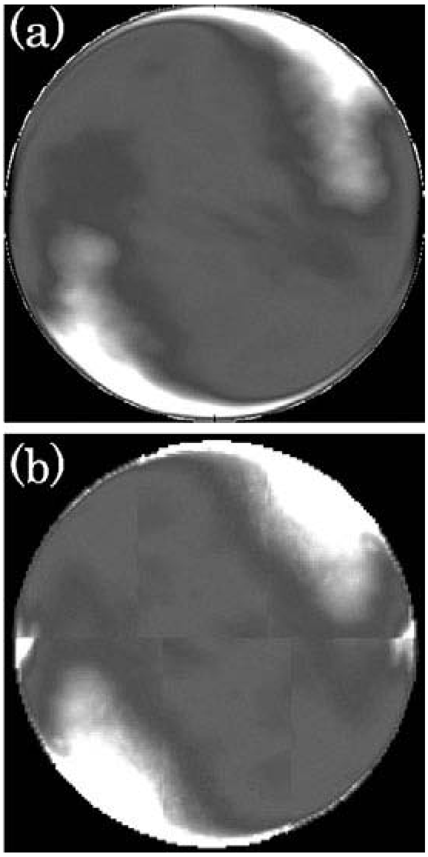

Fig. 1: Turbulence intensity on the central plane perpendicular to the spin axis in the precessing sphere. Reynolds number $Re = 10000$ and the Poincaré number $\mathit{\Gamma} = 0.1$. (a) DNS. (b) Laboratory experiment.