§26. Numerical Analysis of Equilibria of High-beta Toroidal Plasmas with Two-fluid Flow and Pressure Anisotropy

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A numerical analysis of the effects of ion diamagnetic flow and pressure anisotropy on high-beta toroidal magnetohydrodynamic (MHD) equilibria with flow was done by means of the finite element method. A reduced set of Grad-Shafranov (GS) type equilibrium equations for high-beta tokamaks with flow comparable to the poloidal sound velocity, ion finite Larmor radius (FLR), pressure anisotropy and parallel heat fluxes on high-beta tokamaks equilibrium[1)] was used. This set of equilibrium equations consists of the partial differential equations (PDEs) of the first and second order quantities of the magnetic flux. The first order equation is same as that for static equilibria and can be solved iteratively for nonlinear profiles of the lowest order quantities while the second order equation includes contributions from the ion diamagnetic poloidal flows with the gyroviscous cancellation and pressure anisotropy as well as the $E\times B$ flow but is a linear PDE when the solution for the first-order magnetic flux is substituted. The two-fluid effects induce the diamagnetic flows, which result in asymmetry of the equilibria with respect to the sign of the $E\times B$ flow. The circular cross-section and the fixed boundary condition at the normalized minor radius r=1 are assumed. Since the equations for parallel heat fluxes were closed by evaluating the fourth-rank moments with the shifted bi-Maxwellian distribution function and kinetic effects were neglected, there is singularity in the second order equation, where the poloidal flow velocity equals the phase velocity of a sound wave. The profiles of free functions that do not include the vicinity of singularity were chosen to obtain regular solutions.

Figures 1 and 2 show how the isosurfaces of the average pressure p and the ion stream function Ψ, respectively, shift from the magnetic flux surfaces for (a) single-fluid (ideal) MHD, (b) two-fluid MHD with zero Larmor radius (Hall MHD) and (c) two-fluid MHD with ion FLR (FLR two-fluid). Even in the MHD case, pressure is not constant on each flux surface due to flow and pressure anisotropy [Fig. 1 (a)]. On the other hand, Fig. 2 (a) shows that ions flow along the magnetic flux surfaces in the poloidal direction due to the frozen-in law in the MHD case. The shift of the isosurfaces of the ion stream function occurs when the ion diamagnetic flow exists [Fig. 2 (b) and (c)]. The difference between (b) and (c) of Figs. 2 and 3 shows that the FLR effect affects equilibrium and modifies the effect of the ion diamagnetic flow. Figure 3 shows that electrons do not flow along the magnetic flux surfaces only when both of the two-fluid effect and pressure anisotropy exist. We have shown the typical examples of equilibria with the effects of flow, ion FLR and pressure anisotropy.

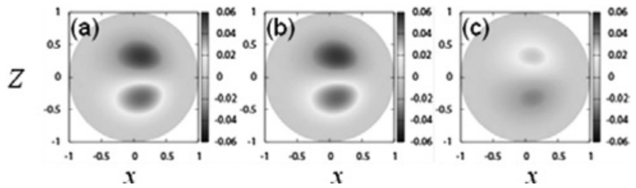

Fig. 1. Profiles of $(\nabla\psi\times\nabla p)\cdot(R\nabla\varphi)$ in the poloidal cross-section for (a) MHD, (b) the Hall MHD and (c) FLR two-fluid models.

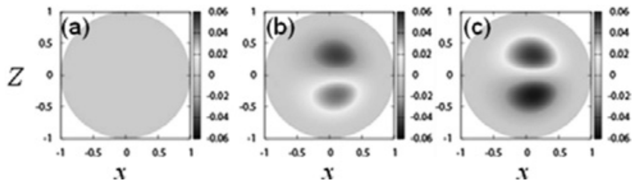

Fig. 2. Profiles of $(\nabla\psi\times\nabla\Psi)\cdot(R\nabla\varphi)$ in the poloidal cross-section for (a) MHD, (b) the Hall MHD and (c) FLR two-fluid models.

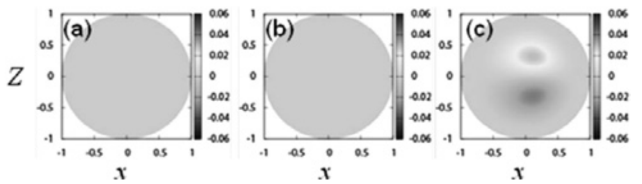

Fig. 3. Profiles of $(\nabla\psi\times\nabla\Psi_e)\cdot(R\nabla\varphi)$ in the poloidal cross-section for (a) MHD with $p_\parallel\neq p_\perp$, (b) FLR two-fluid model with $p_\parallel=p_\perp$ and (c) FLR two-fluid model with $p_\parallel\neq p_\perp$.

1) Ito, A. and Nakajima, N.: Nucl. Fusion **51**, 123006 (2011).