§21. Reduction in the Number of Magnetic Sensors Required for the Reconstruction of the 3D Magnetic Field Profile in the LHD

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1. Introduction

In the Cauchy-condition surface (CCS) method analysis to reconstruct the magnetic field profile in the LHD, an impractically large number of sensors was assumed [1]. To reduce the number of sensors, a possible measure is the reduction in the number of unknowns. We here direct an attention to the singular values in the matrix equation.

In the CCS method, the set of three types of boundary integral equations is discretized and converted to a matrix equation that has the form

$$\mathbf{D}\mathbf{p} = \mathbf{g}\,. \tag{1}$$

The matrix $\mathbf{D}$ is decomposed as $\mathbf{D} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{V}^{\mathrm{T}}$, where $\boldsymbol{\Lambda}$ is a diagonal matrix with non-negative singular value components. The regularized solution is given by

$$\mathbf{p} = \mathbf{V}\boldsymbol{\Lambda}_k^{-1}\mathbf{U}^{\mathrm{T}}\,\mathbf{g}\,, \tag{2}$$

where $\boldsymbol{\Lambda}_k$ means that the singular values smaller than λ_k in $\boldsymbol{\Lambda}$ are omitted so that the condition number (the ratio λ_1 / λ_k) is not larger than a certain value.

2. Behavior of the singular values

Figure 1 shows the singular value behaviors when one assumes 440 field sensors and 126 flux loops. 'T*n*P*m*' means that the CCS is divided into *n* and *m* boundary elements in the toroidal direction and the poloidal direction respectively. The vertical axis represents the singular values whose maximum value is normalized to unity. There is a gap in the vicinity of 10^{-2} in the normalized singular values, independent of the number of boundary elements.

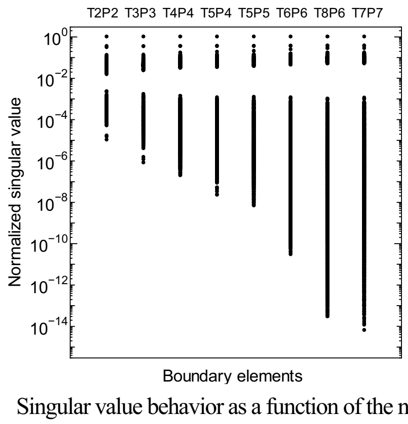

Fig.1 Singular value behavior as a function of the number of boundary elements

The reconstructed field results were compared with the reference solution obtained using the HINT2 code [2]. Figure 2 for the T8P6 case shows (i) the maximum errors of each component of the field and (ii) the portion of the area where the error is larger than 0.02T, as a function of the condition number after the truncation. All the error tendencies were investigated for the region $1.0 < \rho < 1.1$ in the minor radius (ρ) space, i.e. very near the LCMS.

The most accurate results can be obtained when the condition number is 10^2, i.e., when all the singular values smaller than the gap threshold are filtered out.

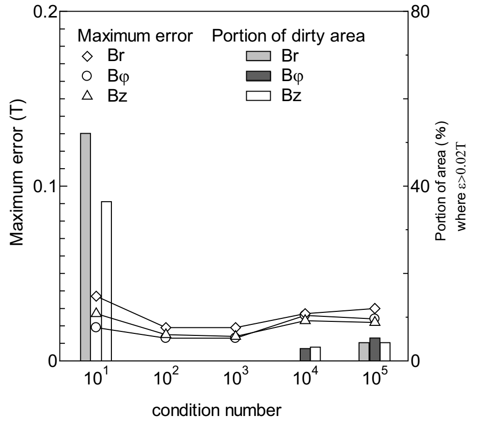

Fig.2 Error tendency of reconstructed field when assuming 440 field sensors and 126 flux loops with T8P6 boundary elements

3. Reduction in the number of boundary elements

We also performed an analysis with only 110 field sensors and 25 flux loops. Following the drastic reduction in the number of sensors we used only 12 boundary elements (T4P3) to enable the analysis.

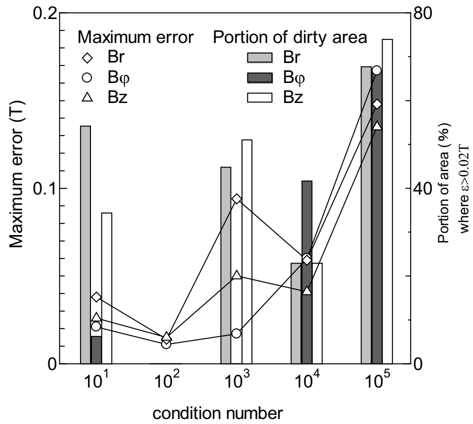

Fig.3 Error tendency when assuming a smaller number of sensors with T4P3 boundary elements

As shown in fig. 3, even in this case the most accurate field results are obtained when the condition number is around 10^2. It should be stressed that the accuracy is higher than that reported in Ref. [1].

4. Conclusion

Accurate results of magnetic field have been obtained even with 110 field sensors and 25 flux loops by cutting off the singular values smaller than the gap threshold.

The number of field sensors can be further reduced to 55 if the symmetry of the field profile is considered. This required number is almost the same as the number of field sensors installed in the LHD, so that present results suggest the possibility of actual application to the LHD.

[1] Itagaki, M., et al. 2012 *Plasma Phys. Control. Fusion* **54**, 125003.
[2] Suzuki, Y., et al. 2006 *Nucl. Fusion* **46** L19.