§28. Turbulent Mixing in a Precessing Sphere

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A rotation of the spin axis of a rotating object is called precession. It is well-known (see Ref. 1 and references therein) that fluid motion in a precessing container becomes turbulent even if the precession is very weak. Since the spin axis of the Earth is precessing with a period of 26000 years, geophysicists have extensively investigated whether or not the Earth's precession affects flow of melted ion in its outer core which generates geomagnetic fields. However, flow in a precessing container is not interesting only from geophysical viewpoints, but also from an engineering point of view. Because strong turbulence may be sustained by a weak precession, this system is likely to have wide variety of applications such as a new type of mixer without impellers, for example.

Towards engineering applications of this system to a mixer, in the present study we have conducted direct numerical simulations (DNS) of turbulence of an incompressible fluid in a precessing sphere, and have estimated its mixing ability. Here, we restrict ourselves within the case that the two rotation axes are perpendicular to each other.

The scheme of the present DNS is as follows. Thanks to the incompressiblity of fluid, its velocity field may be expressed by two scalar functions. Then, governing equations of these scalar functions, which are derived from the Navier-Stokes equations, are numerically integrated by the Crank-Nicolson and Adams-Bashforth methods. In this DNS, spatial derivatives are evaluated by a spectral method, where the scalar functions are expanded in terms of the spherical harmonic functions and the Zernike spherical polynomials. See Ref. 2 for details of this spectral method. In the present study, we track a number of fluid particles advected by the velocity field thus simulated. In this particle tracking, fluid velocity at the position of particles is linearly interpolated, and the temporal integration of particle motion is made by the Adams-Bashforth method.

It is readily shown from the Navier-Stokes equations that flow in a precessing sphere is controlled by two non-dimensional parameters: the Reynolds number $Re = a^2 \Omega_s/\nu$ and the Poincaré number $\Gamma = \Omega_p/\Omega_s$, where a, Ω_s, Ω_p, and ν are the radius of sphere, the magnitude of spin and precession angular velocities, and the kinematic viscosity of fluid, respectively. Our laboratory experiments have shown that turbulence statistics in a precessing sphere drastically change depending on the Poincaré number even for a fixed Reynolds number. Therefore, here we fix the latter parameter at $Re = 40000$ and conducted DNS changing the Poincaré number systematically in a range that $0.01 \leq \Gamma \leq 0.4$.

A temporal evolution of fluid particles in the precessing sphere ($Re = 40000$ and $\Gamma = 0.1$) is shown in Fig. 1, where only the particles in a thin central layer perpendicular both to the spin and the precession axes are plotted. Colors (black and white) of particles are determined due to their initial positions. It is observed that the initially separated two fluids are quite effectively mixed by the turbulence induced by this rather weak precession of the container.

Since the DNS provides us with precise trajectory of each fluid particle, it is possible to estimate mixing efficiency of turbulence. For this quantification, we have employed and modified a simple method originally proposed in Ref. 3. This quantification is based on mixture ratio ρ_i ($i = 1, 2, \cdots, M$) of two fluids in the i-th sub-domain of the container. Here, we divide the sphere into M sub-domains with an approximately same (and sufficiently small) volume. Note that the standard deviation σ of ρ_i tends to be zero as the mixing of two fluids proceeds. We may therefore quantify the degree of mixing in terms of the standard deviation σ.

It is then shown by the present DNS that for the investigated Reynolds number ($Re = 40000$) the most effective mixing takes place when the Poincaré number $\Gamma \approx 0.07$, and that about 10 spins are sufficient to complete the mixing in the precessing sphere for this combination of parameters.

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2) S. Kida & M. Shimizu, *J. Phys.: Conf. Ser.* **318** (2011) 072031.
3) P. V. Danckwerts, *App. Sci. Res.* A **3** (1952) 279.

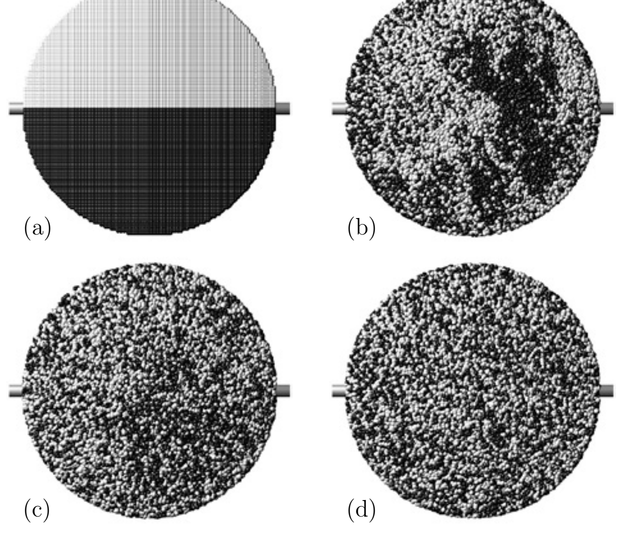

Fig. 1: Turbulent mixing in a precessing sphere. Reynolds number $Re = 40000$ and Poincaré number $\Gamma = 0.1$. A number of fluid particles filled in the sphere are tracked numerically, and only the particles in the central thin layer are visualized. Horizontal and vertical directions are parallel to the spin and the precession axes, respectively. (a) Initial time $t = 0$, (b) $t = 5T_s$ (T_s is the spin period), (c) $t = 10T_s$, (d) $15T_s$.