§21. Performance Measurement of Arbitrary Waveform and Arbitrary Power Factor Matrix Converter

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Quaternion, four-dimensional hyper-complex number, is good at dealing with description of three-dimensional rotation as seen in three-dimensional game graphics programming theory[1]. Utilizing the characteristics, we analyze the phase rotation of three-phase AC of matrix converter, without transforming to two-dimensional rotation in $\alpha\beta$ coordinates.

Now, instead of transforming three-phase AC to two dimension, to represent three-phase AC in three dimension, let's introduce quaternion (four-dimensional hypercomplex number), which is extended from a complex number[2].

$$q = a + iv_x + jv_y + kv_z = a + v \quad (1)$$

$$\begin{cases} i^2 = j^2 = k^2 = -1 \\ ij = -ji = k, jk = -kj = i, ki = -ik = j \end{cases} \quad (2)$$

Quaternion is divided into real part (scalar part) a and imaginary part (vector part) v, similarly with complex number. Namely, vector part has a property of vector, where imaginary numbers i, j, k behave as if they are unit base vectors, but they have also a property of hypercomplex number. To assign three-phase AC to the vector part, let's consider exponential representation of the quaternion.

$$q = a + \hat{n}\|v\| = \|q\|(\cos\theta + \hat{n}\sin\theta) \quad (3)$$

$$\begin{cases} \|q\|^2 = a^2 + \|v\|^2 = a^2 + (v_x)^2 + (v_y)^2 + (v_z)^2 \\ \hat{n} = (+iv_x + jv_y + kv_z)/\|v\| \end{cases} \quad (4)$$

Quaternion can manipulate four dimension, as it is interpreted four-dimension number. But let the scalar part to be zero. When exponential number is multiplied to the vector part from the left-hand side, the vector part rotates by θ in counter clockwise with an axis of the unit vector $\hat{n}$. Here, the rotating axis must be perpendicular to the vector. Generally speaking, we must multiply $\exp(+\hat{n}\theta/2)$ from the left-hand side, and multiply $\exp(-\hat{n}\theta/2)$ from the right-hand side.

Let's assign three-phase AC phase (line-to-neutral) voltages to vector part of quaternion.

$$\begin{aligned} v = \sqrt{2}V\{&+i\cos\omega t + j\cos(\omega t - 2\pi/3) \\ &+k\cos(\omega t - 4\pi/3) = \epsilon^{\hat{n}\omega t}\sqrt{2}VV_0 \end{aligned} \quad (5)$$

$$\begin{cases} \hat{n} = (+i + j + k)/\sqrt{3} \\ V_0 = \{+i\cos 0 + j\cos(-2\pi/3) + k\cos(-4\pi/3)\} \end{cases} \quad (6)$$

Namely, it represents that the initial three-phase (positive phase) AC voltage vector V_0 rotates in counter clockwise with an axis of unit vector $\hat{n}$.

Next, let's consider Ohm's law of three-phase AC circuit, where the load is balanced. Since the neutral impedance (grounding impedance) does not affect for symmetrical (positive phase) AC, the quaternion representation is as follows.

$$v = \epsilon^{\hat{n}\omega t}\sqrt{2}VV_\phi = \{R + p(L - M)\}\epsilon^{\hat{n}\omega t}\sqrt{2}II_0 \quad (7)$$

$$V_\phi = \{+i\cos\phi + j\cos(\phi - 2\pi/3) + k\cos(\phi - 4\pi/3)\} \quad (8)$$

In order to manipulate the three-phase AC similarly as complex vector (phasor) of single-phase AC and symmetrical coordinate method of three-phase AC, let's introduce biquaternion (eight-dimension number), hyper complex number including quaternion.

$$h^2 = -1, ih = hi, jh = hj, kh = hk \quad (9)$$

Namely, we add a complex number h to the quaternion, which is exchangible with quaternion and independent from quaternion. And we assign complex vector representation and the exponential representation of each phase to the complex number h.

$$\begin{aligned} v &= \sqrt{2}V\{+i\epsilon^{h(\omega t+\phi)} + j\epsilon^{h(\omega t+\phi-2\pi/3)} + k\epsilon^{h(\omega t+\phi-4\pi/3)}\} \\ &= \epsilon^{\hat{n}\omega t}\sqrt{2}VV_\phi = \{R + p(L - M)\}\epsilon^{\hat{n}\omega t}\sqrt{2}II_0 \end{aligned} \quad (10)$$

$$V_\phi = \{+i\epsilon^{h\phi} + j\epsilon^{h(\phi-2\pi/3)} + k\epsilon^{h(\phi-4\pi/3)}\} \quad (11)$$

Namely, let's combine the effective value and initial biquaternion into biquaternion, and we can obtain the same representation as the complex vector reprezentation of single-phase AC. And we have only to use unit pure quaternion $\hat{n}$ instead of imaginary number j.

With quaternion, complex power is expressed as follows (Asterisk symbol in superfix means complex conjugate of quaternion).

$$\begin{aligned} vi^* &= (iv_a + jv_b + kv_c)(ii_a + ji_b + ki_c)^* \\ &= (v_a i_a + v_b i_b + v_c i_c) - i(v_b i_c - v_c i_b) \\ &\quad -j(v_c i_a - v_a i_c) - k(v_a i_b - v_b i_a) \end{aligned} \quad (12)$$

Namely, concerning quaternion power, the scalar part means the active power of three-phase (positive phase) and the vector part means the reactive power.

By introduction of biquaternion concept, we can deal with phasor of each phase of three-phase AC similarly as in symmetrical coordinate method. Therefore, not only symmetrical three-phase AC (positive phase) but also negative phase and zero phase can be dealt with. Concerning quaternion power of three-phase AC, we can obtain the similar result as the one in pq theory[3].

Quaternion can not only rotate three-dimensional vector but also divide three-dimensional vector. The characteristics should be utilized to anlyze matrix converter based on space vector method in more detail.

1) E. Lengyel, "Mathematics for 3D Game Programming & Computer Graphics" 2nd ed., *Charles River Media, Inc.*, 2004.
2) J. H. Conway, D. Smith, "On Quaternions and Octonions: Their Geometry, Arithmetic, and Symmetry", *A. K. Perters, Ltd.*, 2003.
3) H. Akagi, Y. Kanazawa, K. Fugita, A. Nanba, "Generalized Theory of the Instantaneous Reactive Power and its Application", *Trans. IEE of Japan*, vol. 103-B, no. 7, pp. 483-490, 1983.