## §27. Magnetic Flux Coordinates for Equilibria with Strong Poloidal Flow

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A system of magnetic flux coordinates are constructed from an analytic solution for the reduced magneto-hydrodynamic (MHD) equations for high-beta toroidal equilibria in the presence of poloidal flow comparable to the poloidal sound velocity. The reduced equilibrium equations were derived by the asymptotic expansions in the inverse aspect ratio $\varepsilon$ of a torus, and include the Grad-Shafranov type equations for the first and second order magnetic fluxes, $\psi_1$ and $\psi_2$[1]. An analytic solution of $\psi \simeq \psi_1 + \varepsilon\psi_2$ was found by assuming circular cross-section, fixed boundary and linear profiles for the flux surface quantities[2]. The solution with second order quantities shows that the magnetic structure is modified and the pressure isosurfaces depart from the magnetic flux surfaces due to the poloidal flow. A system of magnetic flux coordinates is obtained from this analytic solution.

Magnetic flux coordinates consist of a flux surface quantity and an arbitrary poloidal angle in a poloidal cross-section of an axisymmetric torus. The following relations between the coordinates ($x$,$z$) in the cylindrical coordinate system and ($\xi$,$\Theta$) in the magnetic flux one for a poloidal cross-section are adopted,

$$\left(1-\xi^2\cos^2\Theta\right)\left[\psi_{11}(\Delta_s)+\varepsilon\psi_{21}(\Delta_s)\right] = \psi_{11}(x)+\varepsilon\psi_{21}(x), \quad (1)$$

$$\left(-\xi^2\sin^2\Theta\right)\left[\psi_{11}(\Delta_s)+\varepsilon\psi_{21}(\Delta_s)\right] = \psi_{12}(x,z)+\varepsilon\psi_{22}(x,z), \quad (2)$$

$$\psi_1(x,z)=\psi_{11}(x)+\psi_{12}(x,z),$$

$$\psi_2(x,z)=\psi_{21}(x)+\psi_{22}(x,z),$$

$$\left.\frac{d\psi_{11}}{dx}\right|_{x=\Delta_s}+\varepsilon\left.\frac{d\psi_{21}}{dx}\right|_{x=\Delta_s}=0,$$

which are extensions of those for the first order magnetic flux that yield the analytic solution of $[x(\xi,\Theta),z(\xi,\Theta)]$[3]. The upper panels of Fig. 1 show examples of the system of magnetic flux coordinate system ($\xi$,$\Theta$) obtained by solving Eqs. (1) and (2) numerically. The flux coordinates are modified depending on the poloidal flow velocity. Equations (1) and (2) are also solved analytically by expanding ($x$,$z$) up to the first order in $\varepsilon$. These solutions agree well with each other when the poloidal flow velocity is relatively small. The lower panels in Fig. 1 show poloidal profiles on different flux surfaces. The pressure maxima are located in the outer midplane ($\Theta$=0) for sub-sonic poloidal flow and in the inner midplane ($\Theta=\pm\pi$) for super-sonic poloidal flow.

As an application, the flux surface average of the pressure is calculated by using the flux coordinates as

$$\langle p\rangle(\xi)=\int_{-\pi}^{\pi}p(\xi,\Theta)Jd\Theta \Big/ \int_{-\pi}^{\pi}Jd\Theta.$$

The profile of the flux surface average of the pressure is peaked near the magnetic axis for sub-sonic poloidal flow while it is broad for super-sonic poloidal flow, compared with that for the static equilibrium. The stability of high-beta toroidal equilibria with strong poloidal flow in will be studied based on this flux coordinate system.

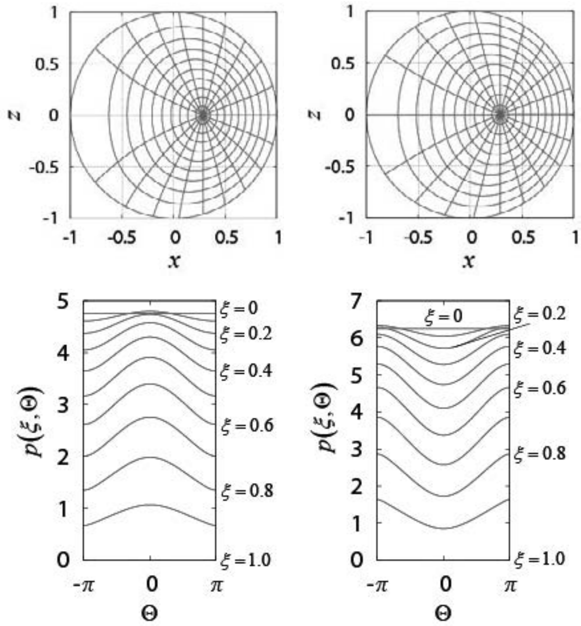


Fig.1. Magnetic flux coordinate systems (upper panels) and poloidal profiles of the pressure on different flux surfaces (lower panels) for sub-sonic (left) and super-sonic (right) poloidal flows.

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2) A. Ito and N. Nakajima, Plasma Phys. Control. Fusion **51**, 035007 (2009).
3) R. Kleibergen and J. P. Goedbloed, Plasma Phys. Control. Fusion **30**, 1939 (1988).