§27. Study on Universality and Singularity of NS and MHD Turbulence at Asymptotic State by Using Massive Parallel Computation

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Huge transport power and strong fluctuations are characteristics of turbulence. Anomalous fluctuations of velocity field drives more the fluctuations and singular nature of the scalar and magnetic field. Fluctuations at large scales are affected by the boundary shape and various conditions so that those statistics are analyzed individually. On the other hand, at small scales of motion where the inertial and viscous terms plays dominant roles, the statistics of fluctuations of the turbulence and scalar are expected to be universal, which in turn are very crucial to constructing the statistical theory of turbulence and subgrid modeling. One of the interesting and difficult characters of fluctuations in turbulence is the fact that the statistical distributions changes from the nearly Gaussian to the strongly non-Gaussian with decrease of scales of motion. The behavior is well quantified in terms of the scaling exponent ζ_n of the structure function $S_n(r) = \langle[\theta(x+r) - \theta(x)]^n\rangle = \langle\delta\theta_r^n\rangle \propto r^{\zeta_n}$. The theories of Kolmogorov and Obukhov-Corrsin predict $\zeta_n = n/3$. However, the observed (or computed) values tend to be smaller than $n/3$ as the order n increases. In the case of passive scalar, the data of ζ_n scatters strongly when compared to the velocity case. This fact suggests that the universality of the scaling exponents of the passive scalar does not exist or at least is weaker than in the case of velocity.

To get an insight into the problem, we have studied the scaling exponents of two passive scalars which are convected by the identical turbulence but excited by different ways by the large scale direct numerical simulations using the plasma simulator. One scalar is excited by the Gaussian random white in time at low wavenumbers and the other is by the term $-\Gamma u_3$ added to the right hand side of the scalar equation which is due to the the mean uniform scalar gradient $d\bar{\Theta}/dz = \Gamma$ imposed in the z direction, respectively. The velocity field is also forced by the Gaissian random white in time at low wavenumbers. The number of grid points is $N = 2048^3$ and the equations of the Naveir-Stokes and two passive scalars are numerically integrated by using the spectral method and 4th order Runge Kutta Gill method at $R_\lambda \approx 500$. Figure 1 shows Kolmogorov's 4/5 law and Yaglom's law

$$\langle\delta u_r^3\rangle = -(4/5)\bar{\epsilon}r, \qquad \langle\delta u_r \delta\theta_r^2\rangle = -(4/3)\bar{\chi}r$$

, where $\bar{\epsilon}$ and $\bar{\chi}$ are the average rates of the energy and scalar dissipations, respectively. It can be seen that the curves divided by $-\bar{\epsilon}r$ or $-\bar{\chi}r$ attain 4/5 or 4/3 level, respectively. The local scaling exponents of the two scalars which are defined as $\zeta_n(r) = d\log S_n(r)/d\log r$ are shown in Figs. 2 and 3, respectively. When the order is low, especially for $n = 2$ both exponents are almost identical, meaning that the spectra of the two scalars are the same. However, for large n, as seen in the figures, the scaling exponents differ considerably at large scales. Indeed, the curves of the scalar 2 are horizontal while the curves of the scalar 1 increase with r. This observation suggests that (1) the universality does not exist at large order, or (2) the Reynolds number is too low to observe the clear scaling exponents. To pin down the second possibility we are conducting the larger DNSs with the finer resolution up to $N = 4096^3$ grid points.

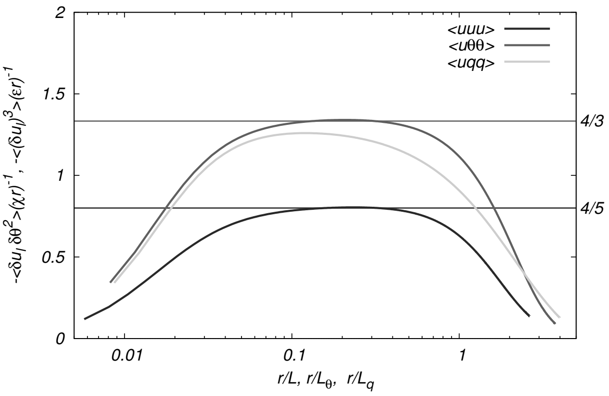

Fig. 1: Kolmogorov's 4/5 law and Yaglom's 4/3 law.

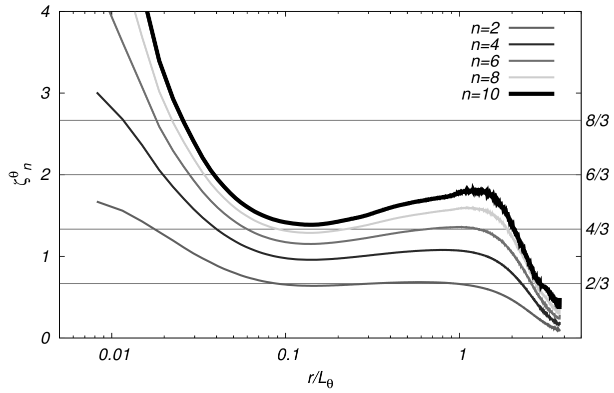

Fig. 2: Local scaling exponents for the random scalar injection at low wavenumbers. $R_\lambda \approx 500$.

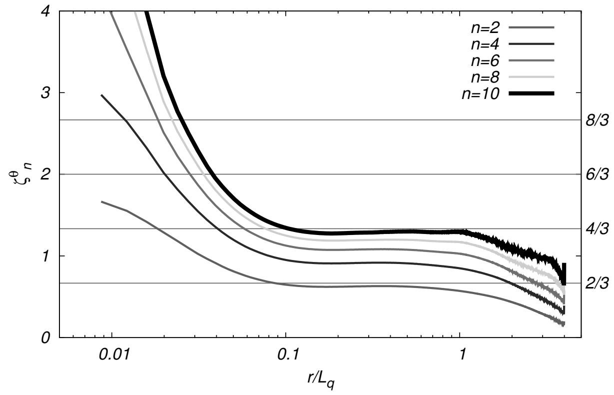

Fig. 3: Local scaling exponents for the scalar injection due to the mean scalar gradient. $R_\lambda \approx 500$.