§29. Hierarchy of Anti-parallel Vortex Tubes in Turbulence

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Small-scale statistics of turbulence are independent of the large-scale information such as the external force or the boundary conditions. This small-scale universality is not only scientifically interesting but also important in engineering applications because it gives a foundation of turbulence models. This remarkable feature is usually explained in terms of the so-called energy cascade: Through the process that energy injected at a large scale cascades down to smaller scales in a scale-by-scale manner, the information of large-scale structures is lost, and the small-scale statistics become independent of it.

Describing the concrete mechanism of the energy cascade has been one of the most important unsolved problems in the field of turbulence. Thanks to the development of numerical environments, we can now tackle this problem by the help of direct numerical simulations (DNS) of turbulence at sufficiently high Reynolds numbers. For example, we proposed [1,2] a scenario of the energy cascade on the basis of the numerical observation that turbulence is composed of a hierarchy of vortex tubes. This implies that the energy cascade is a process in which smaller-scale (i.e. thinner) vortex tubes are created by large-scale (i.e. fatter) vortex tubes. We also observed that vortex tubes tend to align with each other in an anti-parallel manner, and strong strain fields around such anti-parallel pairs create smaller-scale vortex tubes via vortex stretching.

However, the mechanism due to the action of anti-parallel pairs of vortex tubes was suggested on the basis of some examples in our DNS. Therefore, it is still unknown how significant this mechanism is, and what kind of dynamics lead to such anti-parallel pairing. So, in the present study, we investigate the degree of the anti-parallel alignment of vortex tubes on each level of the hierarchy of vortex tubes.

For this purpose, we conducted the DNS of turbulence of an incompressible fluid in a periodic cube. In our DNS, we numerically integrated the Navier-Stokes equation by using the fourth order Runge-Kutta-Gill scheme and the Fourier spectral method, in which the aliasing errors are removed by the phase shift method. We used from 256^3 to 2048^3 Fourier modes depending on the Reynolds number.

In order to capture the hierarchy of vortex tubes, we used a band-pass filter of the Fourier modes of the vorticity field. By numerically solving the Poisson equation, the source term of which was expressed in terms of the coarse-grained flow field, we obtained a coarse-grained pressure field. Then, applying the low-pressure method [3,4], which was originally proposed to identify the Kolmogorov-scale eddies, to this coarse-grained pressure field, we identified the axis of the vortex tubes on an arbitrarily level of the hierarchy. The low-pressure method is a threshold-free scheme to identify vortex tubes based on the fact that the pressure tends to take smaller values on the axis of a vortex tube than its surroundings.

We investigated the distribution of the angle between the vortex axes identified by using the low-pressure method. More concretely, defining short segments of the vortex axes, we calculated the angle, θ, between two segments of two different vortex axes. The probability density function (PDF), $P(\cos\theta|r)$, of the cosine of θ conditioned by their distance r was then evaluated. A result is shown in Fig. 1 for the case that the Taylor length based Reynolds number is about 180 and the coarse-grained length scale ℓ_c is about 40η. Here, η denotes the Kolmogorov length. For distant vortex axes ($3.0 < r/\ell_c < 3.5$), $P(\cos\theta)$ becomes approximately constant indicating isotropic orientations; whereas for nearby vortex axes ($0.8 < r/\ell_c < 0.9$), P has a peak in the vicinity of $\cos\theta = -1$. This implies the tendency that these nearby vortex axes align with each other in an anti-parallel manner.

It is further interesting to observe that this angler distribution, conditioned by r, is independent of the coarse-graining scale ℓ_c and the Reynolds numberR_λ as far as r/ℓ_c is fixed (figure is omitted).

These numeral results show that the vortex tubes on any level of the hierarchy, indeed, tend to align with each other anti-papralelly. Since there exist strongly straining fields around these anti-parallel pairs of vortex tubes, they are likely to create smaller scale vortex tubes effectively. We are investigating the alignment of vortex axes belonging to different levels of the hierarchy to understand the physical mechanism of the anti-parallel alignments (Fig. 1) and to further verify the scenario of the energy cascade in terms of vortex stretching.

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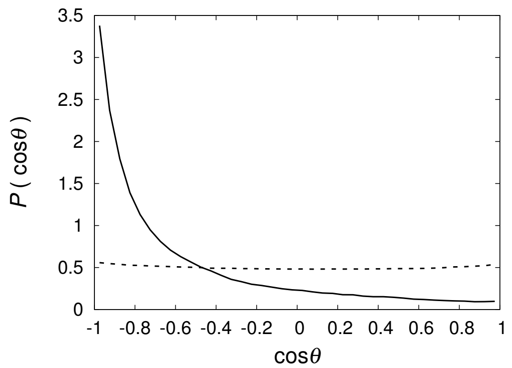

Fig. 1: The PDF of the cosine of the angle θ between segments of vortex axes at the length scale $\ell_c \approx 40\eta$. Solid line, the PDF conditioned by the distance r between the axes as $0.8 < r/\ell_c < 0.9$; dashed line, $3.0 < r/\ell_c < 3.5$.