§35. The Relation of Traffic Flow in One-lane to the Sensitivity in the Stochastic Optimal Velocity Model

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Traffic dynamics has been attracting much attention from physicists, engineers, mathematicians, and economist as a typical example of the system to which non-equilibrium statistical mechanics will be applicable [1),2)].

For the last decade, the mathematical analysis method of the traffic flow in one-lane have been progressed. Especially, since the invention of the Stochastic Optimal Velocity (SOV) model, it is gradually understood that the phase transition phenomenon described by thermodynamics in the traffic flow appears [1)]. It have been found that an emergence of more than one different flux at the same density in the transit region from free phase to congested phase is almost universal in real traffic.

In the general frame work of the stochastic Optimal Velocity model, we prepare a single-lane road which is divided into a one dimensional array of L site. N vehicles are moving on the road and each site contains one vehicle at most, and collision and overtaking are prohibited. In this study all the vehicles attempt to move at each step. We introduce a probability of the *i* th vehicle hopping one site ahead at time t as v_i^t. Then assuming that the next intenstion v_i^{t+1} is determined by v_i^t and the positions of vehicles, x_j^t ($j=i, i+1$), the configuration of vehicles is recursively updated according to the following procedure:

1. For each vehicle, calculate the next intention (probability) to hop forward site with configuration x_j^t ($j=1,\cdots,N$),

$$v_i^{t+1}=(1-a)v_i^t+aV(\Delta x_i^t)\,, \tag{1}$$

   where the sensitivity a and the optimal velocity (OV) function V are restricted to values in $0 \le a \le 1$ and $0 \le V(\Delta x) \le 1$, respectively. The headway distance Δx_i^t is defined as $\Delta x_i^t = x_{i+1}^t - x_i^t + 1$.
2. The probability of the *i* th vehicle stays at x_i^t at next step $t+1$ set to be $1-v_i^t$.
3. Determine the number of sites V_i^{t+1} at which a vehicle moves forward according to the probability v_i^{t+1}.
4. Each vehicle moves avoiding a collision.

$$x_i^{t+1}=x_i^t+\min(\Delta x_i^t, V_i^{t+1})\,, \tag{2}$$

In order to investigate a phenomenological feature, we take OV function as

$$V(x)=(\tanh(x-c)+\tanh c)/(1+\tanh c) \tag{3}$$

where we select c as 3/2. We show in figure 1 the fundamental diagram of the SOV model with $a=0.005$ at $t=1000$, 3000, and 10000, starting from random spacing. Three distinct branches (free-flow, congested, and jam branch) are observed.

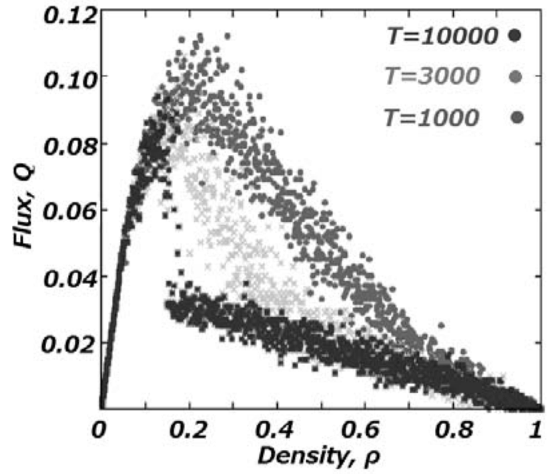

Fig. 1. Fundamental diagram of the SOV model with $a=0.005$. Three branches (free-flow, congested, and jam branch) are observed.

Next we investigated the SOV model with changing the sensitivity a. Figure 2 show the fundamental diagrams of the SOV model at time step $t=10000$ with $a=0.2$, and 0,6, in figure 2(a), and 2(b), respectively.

(a) (b)

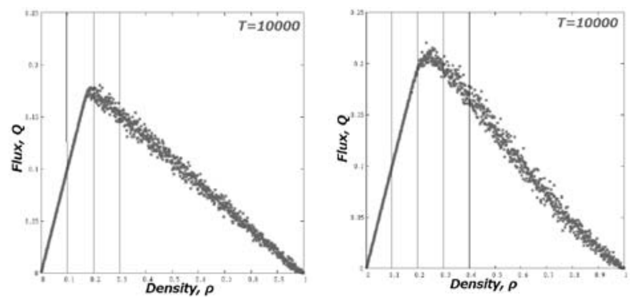

Fig. 2. Fundamental diagram of the SOV model with $a=0.2$ is shown in (a) and $a=0.6$ in (b). The congested branch disappears and the free flow phase and the jam phase are connected continuously.

The free flow phase and the jam phase become connected continuously when a becomes larger than a certain value. As shown above, we found that the fundamental diagram with large sensitivity remarkably differs from that with small one.

1) M. Kanai, K. Nishinari, and T. Tokihiro, *Phys. Rev. E* 72, 035102(R) (2005).
2) Y. Sugiyama, and H. Yamada, *Phys. Rev. E 55*, 7749 (1997).