§16. Performance Measurement of Arbitrary Waveform and Arbitrary Power Factor Matrix Converter

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Quaternion, four-dimensional hyper-complex number, is good at dealing with description of three-dimensional rotation as seen in three-dimensional game graphics programming theory[1]. Utilizing the characteristics, we analyze the phase rotation of three-phase AC of matrix converter.

Now, instead of transforming three-phase AC to two dimension, to represent three-phase AC in three dimension, let's introduce quaternion, which is extended from a complex number[2].

$$q = a + iv_x + jv_y + kv_z = a + v \tag{1}$$

$$\begin{cases} i^2 = j^2 = k^2 = -1 \\ ij = -ji = k, jk = -kj = i, ki = -ik = j \end{cases} \tag{2}$$

Quaternion is divided into real part (scalar part) a and imaginary part (vector part) v, similarly with complex number. Namely, vector part has a property of vector, where imaginary numbers i, j, k behave as if they are unit base vectors, but they have also a property of hypercomplex number. To assign three-phase AC to the vector part, let's consider exponential representation of the quaternion.

$$q = a + \hat{n}\|v\| = \|q\|(\cos\theta + \hat{n}\sin\theta) = \|q\|\epsilon^{\hat{n}\theta} \tag{3}$$

$$\begin{cases} \|q\|^2 = a^2 + \|v\|^2 = a^2 + (v_x)^2 + (v_y)^2 + (v_z)^2 \\ \hat{n} = (+iv_x + jv_y + kv_z)/\|v\| \end{cases} \tag{4}$$

Quaternion can manipulate four dimension, as it is interpreted four-dimension number. But let the scalar part to be zero. When exponential number is multiplied to the vector part from the left-hand side, the vector part rotates by θ in counter clockwise with an axis of the unit vector $\hat{n}$. Here, the rotating axis must be perpendicular to the vector. In general case, we must multiply $\exp(+\hat{n}\theta/2)$ from the left-hand side, and multiply $\exp(-\hat{n}\theta/2)$ from the right-hand side.

Let's assign three-phase AC phase (line-to-neutral) voltages to vector part of quaternion.

$$\begin{aligned} v &= \sqrt{2}V\{+i\cos\omega t + j\cos(\omega t - 2\pi/3) \\ &+k\cos(\omega t - 4\pi/3)\} = \epsilon^{+\hat{n}\omega t/2}\sqrt{2}VV_0\epsilon^{-\hat{n}\omega t/2} \end{aligned} \tag{5}$$

$$\begin{cases} \hat{n} = (+i + j + k)/\sqrt{3} \\ V_0 = \{+i\cos 0 + j\cos(-2\pi/3) + k\cos(-4\pi/3)\} \end{cases} \tag{6}$$

Namely, it represents that the initial three-phase (positive phase) AC voltage vector V_0 rotates in counter clockwise with an axis of unit vector $\hat{n}$.

Next, let's consider Ohm's law of three-phase AC circuit, where the load is balanced. Since the neutral impedance (grounding impedance) does not affect for symmetrical (positive phase) AC, the quaternion representation is as follows.

$$\begin{aligned} v &= \epsilon^{+\hat{n}\omega t/2}\sqrt{2}VV_\phi\epsilon^{-\hat{n}\omega t/2} \\ &= \{R + \hat{n}\omega(L - M)\}\epsilon^{+\hat{n}\omega t/2}\sqrt{2}II_0\epsilon^{-\hat{n}\omega t/2} \end{aligned} \tag{7}$$

$$V_\phi = \{+i\cos\phi + j\cos(\phi - 2\pi/3) + k\cos(\phi - 4\pi/3)\} \tag{8}$$

In order to manipulate the three-phase AC similarly as complex vector (phasor) of single-phase AC and symmetrical coordinate method of three-phase AC, let's introduce biquaternion.

$$h^2 = -1, ih = hi, jh = hj, kh = hk \tag{9}$$

Namely, we add a complex number h to the quaternion, which is exchangible with quaternion and independent from quaternion. And we assign complex vector representation and the exponential representation of each phase to the complex number h.

$$\begin{aligned} v &= \sqrt{2}V\{+i\epsilon^{h(\omega t+\phi)} + j\epsilon^{h(\omega t+\phi-2\pi/3)} + k\epsilon^{h(\omega t+\phi-4\pi/3)}\} \\ &= \epsilon^{+\hat{n}\omega t/2}\sqrt{2}VV_\phi\epsilon^{-\hat{n}\omega t/2} \\ &= \{R + \hat{n}\omega(L - M)\}\epsilon^{+\hat{n}\omega t/2}\sqrt{2}II_0\epsilon^{-\hat{n}\omega t/2} \end{aligned} \tag{10}$$

$$V_\phi = \{+i\epsilon^{h\phi} + j\epsilon^{h(\phi-2\pi/3)} + k\epsilon^{h(\phi-4\pi/3)}\} \tag{11}$$

Namely, let's combine the effective value and initial biquaternion into biquaternion, and we can obtain the same representation as the complex vector representation of single-phase AC. And we have only to use unit pure quaternion $\hat{n}$ instead of imaginary number j.

Concerning equation for generator, we can obtain the biquaternion representation as follows.

$$\begin{aligned} &\epsilon^{h\omega t}\sqrt{2}VV_{0\phi} = 0 \\ &-\{R + h\omega(L + 2M)\}\epsilon^{h\omega t}\sqrt{2}II_{00} \end{aligned} \tag{12}$$

$$\begin{aligned} &\epsilon^{+\hat{n}\omega t/2}\sqrt{2}VV_{1\phi}\epsilon^{-\hat{n}\omega t/2} = \epsilon^{+\hat{n}\omega t/2}\sqrt{2}EE_1\epsilon^{-\hat{n}\omega t/2} \\ &-\{R + \hat{n}\omega(L - M)\}\epsilon^{+\hat{n}\omega t/2}\sqrt{2}II_{10}\epsilon^{-\hat{n}\omega t/2} \end{aligned} \tag{13}$$

$$\begin{aligned} &\epsilon^{-\hat{n}\omega t/2}\sqrt{2}VV_{2\phi}\epsilon^{+\hat{n}\omega t/2} = 0 \\ &-\{R - \hat{n}\omega(L - M)\}\epsilon^{-\hat{n}\omega t/2}\sqrt{2}II_{20}\epsilon^{+\hat{n}\omega t/2} \end{aligned} \tag{14}$$

Attention is necessary to the direction $I_{00}//\hat{n}$. Zero-phase AC does not rotate but does oscillate along the $\hat{n}$. Attention is also necessary to the sign of $\hat{n}\omega$ of the inductive reactance in the negative phase equation.

By introduction of biquaternion concept, we can deal with phasor of each phase of three-phase AC similarly as in symmetrical coordinate method. Therefore, not only symmetrical three-phase AC (positive phase) but also negative phase and zero phase can be dealt with.

Quaternion can not only rotate three-dimensional vector but also divide three-dimensional vector. The characteristics should be utilized to analyze matrix converter based on space vector method in more detail.

1) E. Lengyel, "Mathematics for 3D Game Programming & Computer Graphics" 2nd ed., *Charles River Media, Inc.*, 2004.
2) J. H. Conway, D. Smith, "On Quaternions and Octonions: Their Geometry, Arithmetic, and Symmetry", *A. K. Perters, Ltd.*, 2003.