§6. Development of Implicit MHD Code

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To clarifying the characteristics of pressure driven MHD instabilities for high beta LHD plasmas is one of critical issues. From the previous simulation study [1], it is found that resistive ballooning modes are linearly unstable in the peripheral region using the MIPS code [2] where the fixed boundary condition is imposed at the plasma-vacuum boundary. Since the MHD instabilities are destabilized in the peripheral region, the fixed boundary condition may influence the MHD stability for such high beta LHD plasmas, so that the free-boundary condition may be suitable for such situation. For imposing the free-boundary condition, the pseudo-vacuum model is often used where the vacuum region is replaced with the cold and low density plasma.

In the MIPS code, the fourth-order explicit Runge-Kutta method is used for the time integration. The explicit time integration method has the problem of the CFL condition where the interval of the time step is limited by the fastest physics phenomenon. For extending the MIPS code to treat the free-boundary problem using the pseudo-vacuum plasma model, the initial density profile should be small in the pseudo-vacuum region. In the pseudo-vacuum region, the Alfven velocity increases, so that the problem of the CFL condition becomes increasingly serious. In order to solve this problem, we are developing new MHD code using implicit time integration method. In the implicit time integration method, the interval of the time step is not limited by the CFL condition, so that the simulation can be done using the large interval of the time step. In this study, the linear MHD code with the implicit time integration method has been developed based on the MIPS code. In the implicit MHD code, the fourth-order finite difference method is used for the spatial derivatives and the second-order Crank-Nicolson method is used for the time integration. A large sparse matrix problem in the Crank-Nicolson method is solved using the GRMES method in PETSc library [3]. In order to improve the convergence of the GMRES metod, the physics-based preconditioning (P.B.P.C.) method [4] is introduced. The physics-based preconditioning method can improve the convergence of the GMRES method by transforming the hyperbolic problem into the parabolic problem using the Schur decomposition.

For evaluation of the efficiency of the implicit code, some test simulations have been carried out. For simplicity, the density profile is assumed to be

$$\rho(r,\phi,z)=(\rho_0-\rho_v)(P(r,\phi,z)/P_0)+\rho_v$$

where ρ_0 is the plasma density at the center, ρ_v is the plasma density at the plasma boundary, P is the pressure and P_0 is the pressure at the center. Figure 1 shows the dependence of the cpu time T on the ratio of the plasma density ρ_0/ρ_v. For the explicit code, the cpu time scales as $T\propto(\rho_0/\rho_v)^{1/2}$ since CFL condition is limited by the Alfven time which is proportional to $\rho^{-1/2}$. For the implicit code with P.B.P.C., although the cpu time is larger than that for the explicit code when the plasma density at the edge is large, the implicit code with P.B.P.C. is faster than the explicit code when the plasma density at the edge is extremely small. Thus, the implicit time integration method with P.B.P.C. is effective for the free-boundary simulation.

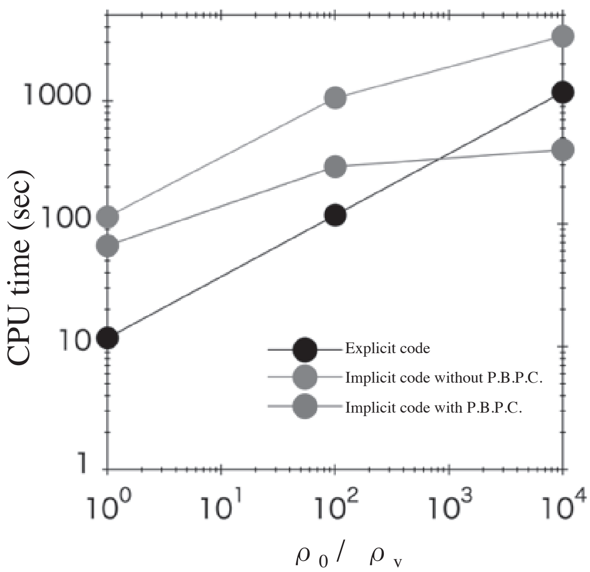

Fig. 1. Dependence of the cpu time on the ratio of the plasma density ρ_0/ρ_v.

1) Sato, M. et al.: Proc. of 24th IAEA Fusion Energy Conf. (Sand Diego, USA, 2012) IAEA-CN-197/TH/P3-25.
2) Todo, Y. et al.: Plasma Fusion Res. **5** (2010) S2062
3) http://www.mcs.anl.gov/petsc
4) Chacon, L., Phys. Plasmas **15** (2008) 056103