§19. Statistical Laws of Strong Fluctuations and Cascade Flux in NS and MHD Turbulent Mixing Studied by Massive Parallel Computation

Gotoh, T. (Nagoya Inst. Tech.)

It is well known that turbulent transport and mixing of scalar such as heat and mass is a key to the understanding and control of the nuclear fusion plasma and the engineering flows. Fluctuations of velocity, scalar, and magnetic fields in the NS and MHD turbulence become stronger with decrease of spacial scale. For example, the moments of scalar difference with the separation distance r are believed to obey a power law $S_n(r) = \langle[\theta(x+r) - \theta(x)]^n\rangle \propto r^{\zeta_n}$. The scaling exponent ζ_n found by the experiments and DNS are increasing more slowly than $n/3$ and have been considered universal. However, recent studies by the experiments, DNSs and theoretical analysis of the Kraichnan model for the passive scalar suggest that the universality of the scaling exponent of the passive scalar in turbulence is not as robust or universal as in the case of the velocity.

We have conducted large scale simulations of two passive scalars which are simultaneously convected by the same turbulent flow in a steady state. Scalar θ is excited by the Gaussian random injection that is white in time and applied to the low wavenumber band, the scalar q is excited by the mean uniform scalar gradient $d\bar{\Theta}/dz = \Gamma$. The velocity field is also excited by the random solenoidal force which is in the same way as in the case of the scalar θ. A set of equations are integrated by the spectral method. The number of grid points are 4096^3 and the Taylor microscale Reynolds number in steady state is about 805. It is very essential to obtain a statistically well converged statistics. The length of the time average is more than $8.2(L/u_{\rm rms})$ which is the longest run than any DNSs ever before at this large number of grid points.

Figure 1 shows the compensated spectra of the two scalars in the Kolmogorov units. The horizontal parts correspond to the Kolmogorov, Obukhov-Corrsin spectrum. The universal constants are found to be $K = 1.83$ (red), $C_\theta = 0.725$ (blue for θ) and $C_q = 0.699$ (green for q) and consistent with the values in the literature. Figure 2 indicates the local scaling exponents defined by $\zeta_n^\alpha(r) = d\log S_n^\alpha(r)/d\log r, (\alpha = q, \theta)$. For $n = 2$, they almost collapse but for larger n they differ at scales in the inertial range. Surprising observation is the fact that ζ_n^θ is a linear function of r in the inertial range, which in turn means that the moments are

$$S_{n0}^\theta(r) = A_{n0}\left(\frac{r}{\overline{\eta}}\right)^{\xi_n^\theta} \exp\left(\beta_n\left(\frac{r}{\overline{\eta}} - 1\right)\right), \quad \beta_n > 0$$

where $\overline{\eta}$ is the mean Kolmogorov length. It is found that β_n is a linear function of n. These findings mean that there is no universality in the scaling exponent of the passive scalar and the power law appears as the prefactor followed by the exponential. On the other hand, the moments of the scalar q obey the power law. Certainly these findings need explanation which is the great challenge to the turbulence theory.

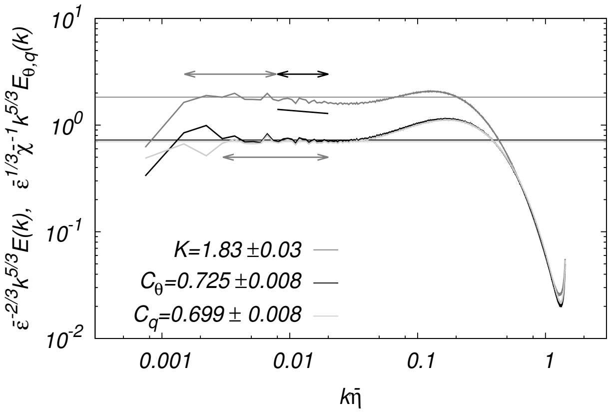

Fig. 1: Compensated spectra of $E(k)$, $E_\theta(k)$ and $E_q(k)$ at $R_\lambda = 805$. The horizontal lines are at $K = 1.83$, $C_\theta = 0.725$ and $C_q = 0.699$, respectively. Short line and arrows show the slope -0.08 and the wavenumber ranges over which the above constants and slope are evaluated, respectively.

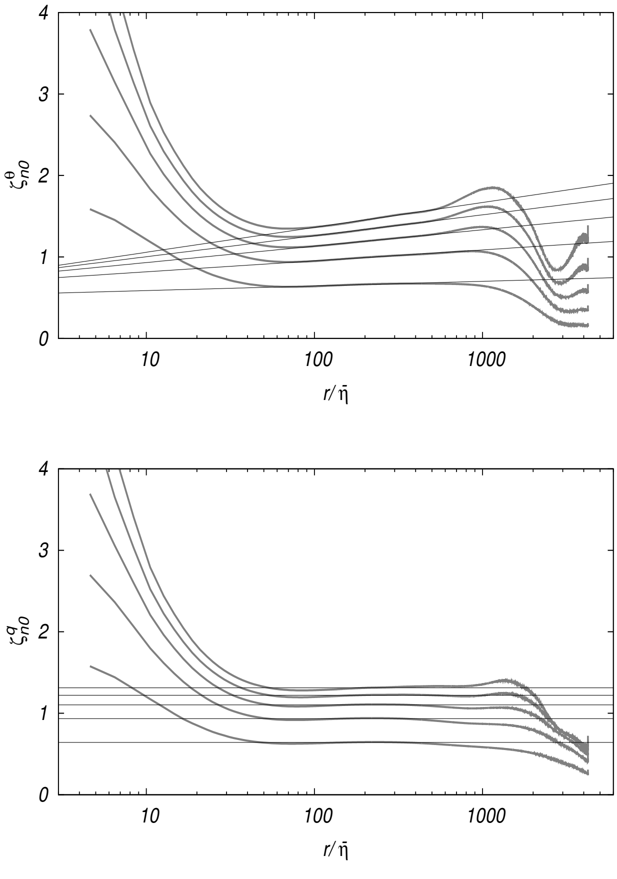

Fig. 2: Local scaling exponents of $\zeta_{n0}^\theta(r)$ (upper) and $\zeta_{n0}^q(r)$ (bottom) for q and $n = 2, 4, 6, 8, 10$.