§21. Development of Highly Accurate Numerical Methods for the MHD Equations and the Full Two-fluid Equations

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Macroscopic dynamics of plasmas is well described by a system of fluid equations with Maxwell's equations. Since, in particular, non-dissipative fluid equations such as the ideal MHD equations and the non-viscous full two-fluid equations are hyperbolic partial differential equations, discontinuous solutions as shock waves may develop within finite time even if the initial conditions are smooth. Actually, in space and astrophysical plasmas, strong shock waves are ubiquitously produced and complicatedly interacted with each other. Thus, it is thought that the discontinuities play an essential role as a basic physical process for various space and astrophysical phenomena.

Higher-order shock capturing schemes, which are able to exactly solve complicated interactions between the shock waves and the turbulence, have been intensively studied in the field of computational fluid dynamics (CFD). Particularly in space and astrophysical fluid problems, first-order approximate Riemann solvers such as Roe's solver and Harten-Lax-van Leer's (HLL) type solvers are extended to higher-order ones using well-established finite volume interpolations: monotone upstream-centered scheme for conservation laws (MUSCL), piecewise parabolic method (PPM), and so on. However, higher-order numerical solutions may not be obtained in real physical simulations because multi-dimensional finite volume method cannot readily achieve more than third-order accuracy in general.

The final objective of this study is to develop a highly accurate multi-dimensional numerical scheme for plasma fluid equations: especially the MHD equations and the full two-fluid equations. In particular, this paper is devoted to investigating basic performance of a non-oscillatory higher-order finite difference method for the one-dimensional MHD equations as a first step.

We adopt a specific approach, so-called weighted compact nonlinear scheme[1)] (WCNS), as a base scheme of the finite difference method. Particularly, the following WCNSs with different order of accuracy are considered:

WCNS-MD5:

$$f_i' = \frac{75}{64}\frac{f_{i+1/2}^* - f_{i-1/2}^*}{\Delta x} - \frac{25}{384}\frac{f_{i+3/2}^* - f_{i-3/2}^*}{\Delta x} + \frac{30}{640}\frac{f_{i+5/2}^* - f_{i-5/2}^*}{\Delta x}$$

WCNS-MND5:

$$f_i' = \frac{3}{2}\frac{f_{i+1/2}^* - f_{i+1/2}^*}{\Delta x} - \frac{3}{10}\frac{f_{i+1} - f_{i-1}}{\Delta x} + \frac{1}{30}\frac{f_{i+3/2}^* - f_{i-3/2}^*}{\Delta x}$$

WCNS-MND4:

$$f_i' = \frac{4}{3}\frac{f_{i+1/2}^* - f_{i+1/2}^*}{\Delta x} - \frac{1}{6}\frac{f_{i+1} - f_{i-1}}{\Delta x}$$

Here numerical flux $f_{i+1/2}^*$ is computed using the left/right variables at the midpoint, which are interpolated from those at adjacent nodes. In the present study, fifth-order nonlinear weighted interpolation function is used. For comparison we also perform numerical experiments by MUSCL and fifth-order finite-volume weighted essentially non-oscillatory (FV-WENO) scheme[2)].

Propagation of the circularly polarized Alfven wave was examined. Though the order of accuracy of WCNS-MND4 is ideally less than that of the other fifth-order schemes, we found that WCNS-MND4 provides almost the same order of accuracy as WCNS-MND5 and fifth-order FV-WENO as seen in Fig.1. On the other hand, WCNS-MD5 might give slightly higher resolution in some cases.

Moreover MHD shock tube tests were performed. We confirmed from Fig.2 that WCNSs are able to sharply capture MHD discontinuities almost as same as WENO. It seems, however, that small numerical oscillations ahead and behind the discontinuities are apparent in WENO rather than in WCNSs though the situation might depend on various parameters in general.

Through several numerical experiments, we concluded that WCNSs can be applied for the plasma fluid equations as a higher-order shock capturing scheme. However, a straightforward extension to multi-dimensions is problematic since the numerical schemes for the plasma fluid equations must preserve Gauss's law, which is quite different from "neutral" hydrodynamics without electro-magnetic fields. Therefore, a multi-dimensional divergence-free finite difference shock capturing scheme is under investigation at present..

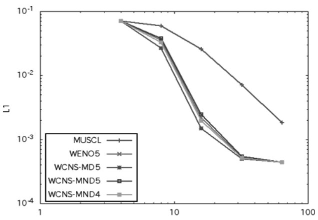

Fig.1. Numerical errors between the numerical solutions and the exact solutions of the nonlinear Alfven wave (L1-norm) versus number of grids.

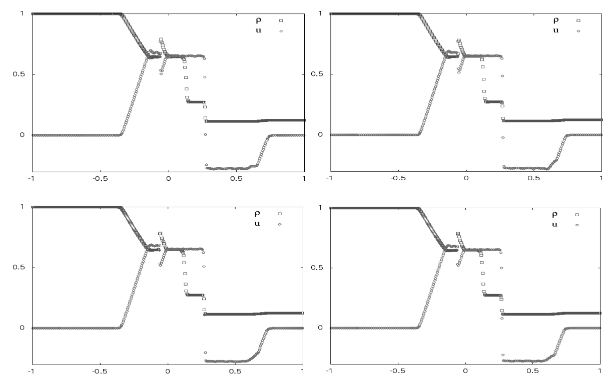

Fig.2. Brio-Wu's MHD shock tube tests by (top-left) fifth-order FV-WENO, (top-right) WCNS-MD5, (bottom-left) WCNS-MND5, (bottom-right) WCNS-MND4, respectively.

1) Nonomura, T., Fujii, K.: Comput. Fluids, **85** (2013) 8.
2) Jiang, C-S., Shu, C-W.: J.Comput. Phys., **126** (1996) 202.