§28. Development of Solver for Large Non-linear Differential Equation and their Engineering Application

Ikuno, S. (Tokyo Univ. of Technology), Nakamura, H.,
Kamitani, A., Saitoh, A. (Yamagata Univ.),
Itoh, T. (Nihon Univ.),
Takayama, T. (Yamagata Univ.),
Hano, N. (Univ. Tokyo),
Nagao, Y. (JCHO Osaka Hospital)

i) Introduction As is well known that an initial and boundary value problem of partial differential equation is obtained from formulating a physical and engineering phenomena. In the electromagnetic behavior simulation of shielding current density in a High Temperature Superconductor (HTS), the time-dependent integro-differential equation should be numerically calculated. After discretizing the system by use of Finite Element Method (FEM) or Meshless method and the completely implicit method, we obtain a nonlinear system. However, improper integrals appear as coefficients of the system and the integrand have a stronger singularity with decreasing thickness of the HTS, and it takes much CPU time to solve the system [1)].

On the other hand, the large scale linear system is appeared in discredizing process of the problem, and it also takes much CPU time to solve the linear system. For this issue, we have proposed the mixed precision Variable Preconditioned Krylov subspace method.

The main purpose of the present study is to evaluate numerically the performance of Variable Preconditioned Krylov subspace method for solving the linear system obtained by various physical and engineering phenomena simulation. In addition, a large-scale simulation of electromagnetic wave propagation using meshless method is also investigated.

ii) Variable Preconditioned Krylov Subspace Method The variable preconditioned GCR method has the sufficient condition for convergence. The residual of the problem converges if the relative residual norm of inner-loop satisfies the inequality in each steps. That is to say, residual equation can be solved roughly by using some iterative method with only a few iteration, and a stationary iterative method such as Gauss-Seidel method, is adopted for variable preconditioning procedure, generally.

The sufficient condition of VPGCR leads us that the residual equation for the preconditioned procedure of VPGCR can be solved in the range of single precision. From this results, we have proposed variable preconditioned GCR method with mixed precision that uses single precision operation for inner-loop and double precision operation for outer-loop. Thus, we extend the algorithm of variable preconditioned method using various Krylov subspace method for outer-loop. Note that the convergence theorem of VPGCR cannot be adopted for the variable preconditioned Krylov subspace method with CG or the Krylov subspace method based on Lanczos principle for outer-loop.

iii) Results and Discussions We have implemented the variable preconditioned Krylov subspace method that inner-loop solver is Jacobi Over-Relaxation method and outer-loop solver is conjugate gradient method on GPU, and have evaluated the performance of the method. The target linear system is obtained from a non-linear magnetostatic field by finite element method with edge elements. The value of number of element N_{elem} is $N_{\mathrm{elem}} = 27,549,822$ and the dimension size of the coefficient matrix of the linear system is $N = 1,709,028$. The result of the computation shows that the mixed precisioned VPCG with JOR on GPU is 41.853 times faster than that of VPCG with CG on CPU.

Next, we have investigated the large-scale simulation of electromagnetic wave propagation using meshless method [3, 4)]. In this study, we have proposed the shape function generation scheme for the meshless method by using multi-core CPU to reduce the searching time of the nodes that belonging inside of influence domain. By using the scheme, CPU time of proposed scheme is overwhelmingly faster than that of ordinal searching scheme. Actually, the CPU time of proposed scheme with OpenMP is about 7842 times faster than that of ordinal scheme without OpenMP. Moreover, we also parallelized the time evolution process using GPU of the wave propagation simulation by meshless method. The result of computation shows that the speedup of CPU time has been achieved more than seven times by using GPU. In addition, the large-scale calculation can be executed [2)].

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