# §29. Analysis of the Rail Network with Properties of Complex Network

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Networks such as railways, human relationships, social networking services and etc. can be found in real world. They have properties of complex network. In this study, we adopt up the railways network spread in Tokyo as an example of the complex network and we analyzed this network by mainly considering three important properties in network theory (scale-free property, small-world property and clustering property) [1,2].

Railways network in the real world is simplified by replacing the stations and railways by nodes and links connecting the stations, respectively. This simplification enables the theoretical analysis of the complex network. We investigate railways network, including 673 stations, which consists of all routes in Tokyo (Figure 1).

The complex network is characterized by following three quantities.

1. Scale-free property
   Distribution of degree $k_i$ follows power law. $p(k) \propto k^{-\gamma}$ , where $k_i$ is the number of the link from node $v_i$ . $\gamma$ is the power exponent.
2. Small-world property
   The average node distance of the whole network $L$ has upper limit, i.e. $L < \log n / \log \langle k \rangle$, where $n$ is total number of node [2].
3. Clustering property
   Cluster coefficient $C$ is defined as the number of the triangles which is formed out of 3 nodes connected by links. This quantity is the index which indicates the degree of concentration of the network connection. It is expected that $C$ satisfies the relation $C > \langle k \rangle / n$ in the complex network.

In figure 2, the distribution of degree $k_i$ of railways in Tokyo ( $n = 673$ ) is plotted. In figure 2(b), it is shown in logarithmic plot. We find that the power index is $\gamma = 2.626$ . $\gamma$ is calculated using least-square method where the points $k = 1,2,3$ in figure 1 are excluded. We calculate the average node distance $L$ and the cluster coefficient $C$ , respectively, $L = 9.74$ , $C = 9.688 \times 10^{-2}$ , however in the calculation of $C$ the point $k = 1$ is excluded. From these result the relations $L > \log n / \log \langle k \rangle$ and $C > \langle k \rangle / n$ are derived.

We studied property of the railways network in Tokyo. The scale-free property and the clustering property were confirmed. On the other hand, the railways network in Tokyo did not follow the small-world property. This indicated that the stations where only local trains stop were overwhelmingly.

1) R. Albert, A.-L. Barabási. Statistical mechanics of complex networks. Review of Modern Physics, **74**(2002) p.47--97.
2) A.-L. Barabási, Linked: The New Science Of Networks Science Of Networks. (Basic Books, United States, 2002) 280.

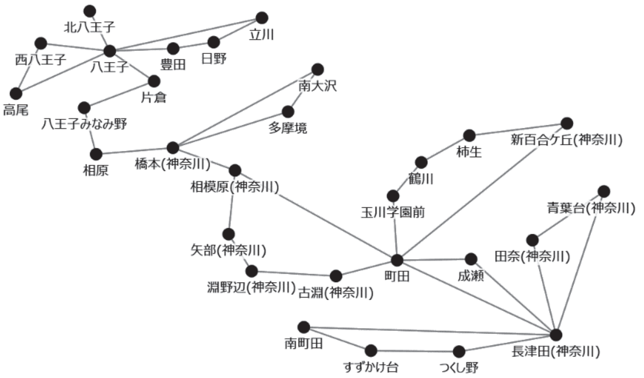


Fig.1 Network in JR Yokohama Line and its circumference

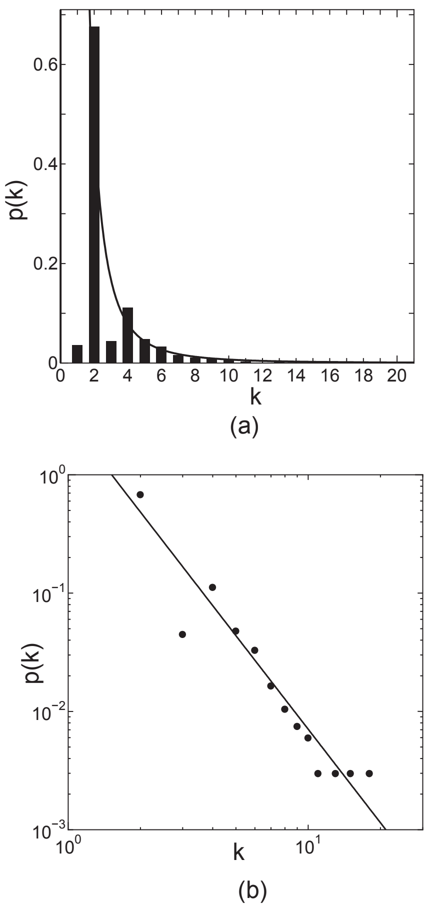


Fig. 2 Distribution of degree $k_i$ .