§7. Mathematical Study of Inverse Problem Solvers for Imaging Science

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The inverse problem theory is mathematical physics in scientific measurements. A physical model of measurement that relates the acquired detector outputs to the object leads to an equation to be solved for unknown parameters of the object. The success is obtained on an appropriate modeling, an excellent solution of the equation, and a good calibration of detector outputs. An important problem in imaging science is the upsizing in numerical inversion. Typical examples are the computed tomography in electron microscopy and adaptive optics, the image synthesis for radio telescopes, the microwave tomography for medical purposes, and also the 3D tomographic imaging of plasma in fusion research. Development of numerical techniques is awaited on both sides of the fast stable iterative solvers and the upsized direct solvers [1, 2].

Study is in progress on iterative solvers of the Tikhonov regularization with the criterion of minimum GCV (generalized cross validation), which is the standard method of inversion, and also on the approximate singular value decomposition (SVD) of a large matrix. Interest is retained on the nonparametric technique of compressed sensing for higher resolution and the parametric technique of Laplacian eigen function expansion for imaging objects having complicated boundaries. In addition, after the plasma imaging applications of small size, investigation is being advanced on possible extensions by adopting the weighted least squares, the generalized SVD, and the distance minimization with use of theoretical phantoms.

A result of the direct solver study is shown in Fig. 1. When the Laplacian matrix is adopted to the penalty function in the Tikhonov regularization (Tikhonov-Phillips regularization) in order to straightforwardly evaluate the smoothness of the object profile, one may have a difficulty in calculating the inverse of Laplacian matrix before executing the SVD. As the inverse problems are upsized, the Laplacian matrix becomes ill-conditioned. A countermeasure is to use the Cholesky decomposition. A calculative success was obtained in the 3D tomography using 4 infrared video-bolometers in LHD [3]. The size of inversion is fairly large with objective voxels and projection data as many as 16,188 (number of unknowns) and 3,196 (number of equations), respectively. And the related projection matrix has a condition number of 3.96×10^6. The inversion was achieved in high accuracy for the right triangular matrix, and the reconstructed image was reasonable. Owing to the adoption of a 3D Laplacian matrix, noisy artifacts were greatly decreased. It is seen in Fig. 1 (b) that the emissivity distribution narrow in poloidal planes is broadened, and that the bright points vary their positions helically in the toroidal direction with changes of brightness. The apparent disconnection in the distribution is related to the voxel size in digitalizing. The effect of the Laplacian matrix, which leads to the improvement of statistical accuracy with a loss of spatial resolution was obtained as expected.

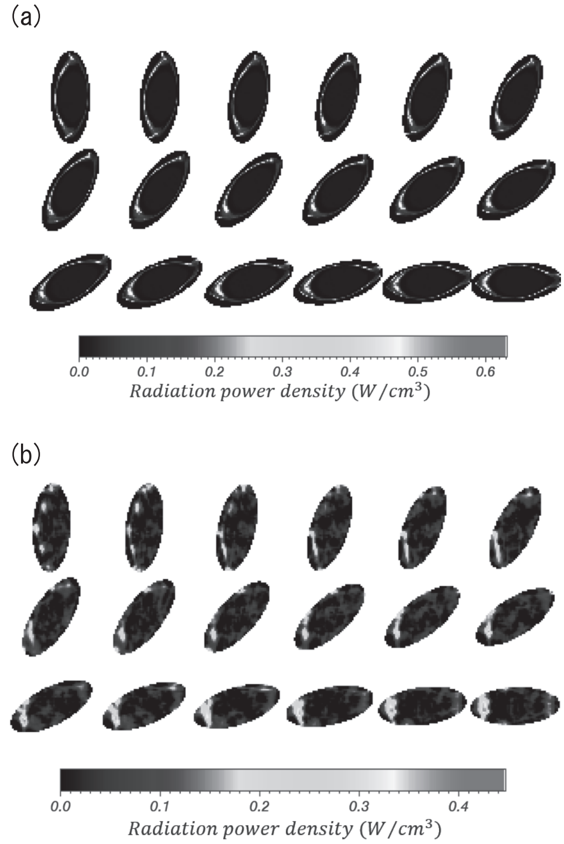

Fig. 1. Result of a numerical simulation of 3D tomography: (a) impurity radiation phantom obtained with the simulation code EMC3-EIRENE; (b) image reconstructed with the aid of Cholesky decomposition and optimized with the criterion of minimum GCV. The 3D profile of helical plasma in LHD is displayed with a series of its poloidal sections, which are lexicographically arranged over the toroidal angle interval from 0.5° to 17.5° with an equal space of 1°.

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1) Iwama, N. et al.: Ann. Rep. NIFS (2012-2013) 472.
2) Iwama, N. et al.: Ann. Rep. NIFS (2013-2014) 465.
3) Iwama, N. et al.: Ann. Rep. NIFS (2013-2014) 191.