§9. Euclid Distance Minimization for Improving the 3D Plasma Image Reconstruction in LHD

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Image reconstruction by solving an underdetermined linear equation can be improved by using prior information on objects. In a plasma imaging experiment, one has some prior knowledge on the objective plasma structure; for example, one predicts the position and broadening of the strong emissivity region. Particularly, in 3D tomography using 4 IRVB's (InfraRed imaging Video Bolo-meters) [1], the projection data are acquired in the form of 2D image in each pinhole camera frame. Also, in today's nuclear fusion experiments, theoretical calculation by computer is well developed and gives plasma profiles that can be referred to.

In the linear regularization of Tikhonov-Phillips, a reference image can be adopted easily in its formulation. In a full algebra expression $H\boldsymbol{f}=\boldsymbol{g}$ with the projection matrix H, the image vector $\boldsymbol{f}$ and the data vector $\boldsymbol{g}$, it is possible to use a reference image $\boldsymbol{f}_{\mathrm{ref}}$. Provided that the sum of squared residuals $\|H\boldsymbol{f}-\boldsymbol{g}\|^2$ is equal to a constant, we require that the object image $\boldsymbol{f}$ should maximally approach the reference $\boldsymbol{f}_{\mathrm{ref}}$. Then, we take a Lagrange function

$$\Lambda(\boldsymbol{f}) = \alpha d(\boldsymbol{f}, \boldsymbol{f}_{\mathrm{ref}}) + \|H\boldsymbol{f}-\boldsymbol{g}\|^2 \qquad (1)$$

to be minimized for a solution $\boldsymbol{f}$. Here, $d(\boldsymbol{f}, \boldsymbol{f}_{\mathrm{ref}})$ is a quantity that represents a distance between $\boldsymbol{f}$ and $\boldsymbol{f}_{\mathrm{ref}}$. The Lagrange multiplier α (>0) is a parameter for adjusting the balance between $d(\boldsymbol{f}, \boldsymbol{f}_{\mathrm{ref}})$ and $\|H\boldsymbol{f}-\boldsymbol{g}\|^2$.

For $d(\boldsymbol{f}, \boldsymbol{f}_{\mathrm{ref}})$, one may take the Kullback-Leibler distance to get a positive value assurance of the image by nonlinear optimization. When one takes the squared Euclid distance $\|\boldsymbol{f}-\boldsymbol{f}_{\mathrm{ref}}\|^2$, it is possible to stay in the linear framework of Tikhonov. Actually, we have a slightly modified solution of the finite series expansion:

$$\boldsymbol{f}(\alpha) = \sum_{m=1}^{M} w_m(\alpha) \frac{\boldsymbol{u}_m^T(\boldsymbol{g} - H\boldsymbol{f}_{\mathrm{ref}})}{\sigma_m} \boldsymbol{v}_m + \boldsymbol{f}_{\mathrm{ref}} \qquad (2)$$

Here, $\{\boldsymbol{u}_m\}$, $\{\boldsymbol{v}_m\}$ and $\{\sigma_m\}$ are two orthonormal systems and the singular values, respectively, that are obtained by the singular value decomposition of the matrix H. The coefficients $w_m(\alpha) = 1/(1+\alpha\sigma_m^{-2})$ are the weights for tapering noisy terms in an ill-conditioned solution. M is the number of the observed projection values.

Eq. (2) means that, to get a reconstruction, we previously subtract the projection of $\boldsymbol{f}_{\mathrm{ref}}$ from the data $\boldsymbol{g}$. Then, we calculate the Tikhonov solution from the remaining parts of the data. Finally, we add the reference image $\boldsymbol{f}_{\mathrm{ref}}$ to accomplish the reconstruction. Only a slight modification of computing code is enough for this calculation. The GCV of the original form is available for α-optimization. Differential operators for profile smoothing can also be adopted if necessary.

The Tikhonov method so modified has been applied to the 3D tomography. The reference image $\boldsymbol{f}_{\mathrm{ref}}$ is synthesized with a parametric function model of the 3D helical object. The function has 8 parameters to be estimated and can give a rough approximation to the emissivity profile that changes temporarily during the radiation collapse. The parameter estimation is achieved by least squares fitting the model, in its projection, to the observed image of a single IRVB. Fig. 1 shows two typical references, $\boldsymbol{f}_{\mathrm{ref}}$, before and after a radiation collapse. Compared to the EMC3-EIRENE phantoms [1], their profiles are reduced in spatial resolution in order to avoid an excessive fidelity to the theoretical prediction. Results of the reconstruction are shown in Fig. 2. The effect is apparent. Noisy artifacts diminish and the profiles of edge and core radiations become clearer. The criterion of minimum GCV worked well. At the right edge of the ϕ=17.5° section, the bulk artifact that appears due to a lack of crossing sightlines still remains. More improvement of the image reconstruction method is desired.

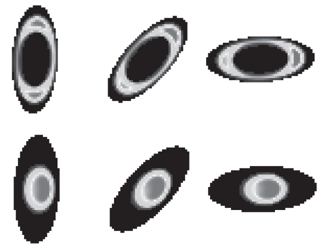

Fig. 1 3D reference image $\boldsymbol{f}_{\mathrm{ref}}$ that is synthesized adaptively for the edge radiation before collapse (upper row) and for the core one after collapse (lower row). Profile is shown in section at poloidal angles ϕ=0.5°, 9.5°, 17.5°; Shot No.121787; 6.0, 6.4 [s].

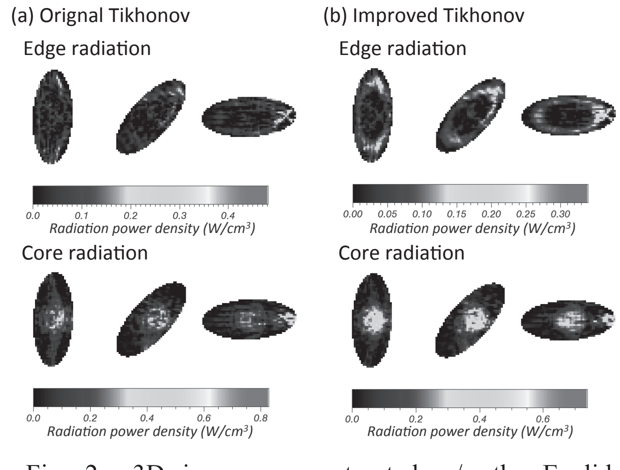

Fig. 2 3D images reconstructed w/o the Euclid distance minimization. ϕ=0.5°, 9.5°, 17.5°; Shot No. 121787; 6.0, 6.4 [s]

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1) Sano, R. et al.: Rev. Sci. Instrum. **87**, 053502 (2016).
2) Sano, R. et al.: 25th Int'l Toki Conf. P1-90 (Nov. 2015, Toki).