§5. Method of Mode Content Analysis Using Measured Signals by a Millimeterwave Beam Position and Profile Monitor

Shimozuma, T., Kobayashi, S., Ito, S., Ito, Y., Kubo, S., Yoshimura, Y., Igami, H., Takahashi, H., Mizuno, Y., Okada, K., Mutoh, T.

The obtained signals in the Beam Profile Monitor (BPM) present the peak position and the intensity profile of the propagating wave in the evacuated corrugated waveguide in almost real-time. As a next step, using the signals obtained by the BPM, a method of mode content analysis is considered and proposed according to the method in the reference[1]. For simplicity, linear polarized modes in a circular corrugated waveguide are considered. The electric fields of the LP-(e) and -(o) modes, in which (e) and (o) mean even and odd, respectively, are expressed as the following equations[1, 2]:

$$\mathrm{LP}^{\mathrm{y}}_{\mathrm{nm}}(\mathrm{e}) : \boldsymbol{E}_{\perp\sigma}(r,\phi) = \hat{y}\sqrt{2} f_\sigma J_n(X_\sigma \cdot r/a)\cos(n\phi), \tag{1}$$

$$\mathrm{LP}^{\mathrm{y}}_{\mathrm{nm}}(\mathrm{o}) : \boldsymbol{E}_{\perp\sigma}(r,\phi) = \hat{y}\sqrt{2} f_\sigma J_n(X_\sigma \cdot r/a)\sin(n\phi), \tag{2}$$

$$\mathrm{LP}^{\mathrm{x}}_{\mathrm{nm}}(\mathrm{e}) : \boldsymbol{E}_{\perp\sigma}(r,\phi) = \hat{x}\sqrt{2} f_\sigma J_n(X_\sigma \cdot r/a)\cos(n\phi), \tag{3}$$

$$\mathrm{LP}^{\mathrm{x}}_{\mathrm{nm}}(\mathrm{o}) : \boldsymbol{E}_{\perp\sigma}(r,\phi) = \hat{x}\sqrt{2} f_\sigma J_n(X_\sigma \cdot r/a)\sin(n\phi), \tag{4}$$

where $\hat{x}$ and $\hat{y}$ are unit vectors in x and y directions, n and m represent the mode numbers and X_σ is the eigen value of the mode σ with (n, m). a expresses the radius of the waveguide, and the normalization constant f_σ is

$$f_\sigma = \frac{\sqrt{Z_0}}{a\sqrt{\pi}J_{n+1}(X_\sigma)} = -\frac{\sqrt{Z_0}}{a\sqrt{\pi}J_{n-1}(X_\sigma)}. \tag{5}$$

The direction of the electric field is oriented to Y-direction.

Generally, a propagating millimeter-wave in a corrugated waveguide is expressed as a superposition of several eigen modes σ $(= 0 \cdots N)$. It is assumed that the miterbends are installed at $L = L_k (k = 0, 1, 2, \dots)$, where L is the distance along the waveguide axis, and L_k denotes the distance between the origin, O, and the center of the k-th miterbend reflector. The BPM system consists of two BPMs installed in the 90-degree miterbends(θ = 45deg.), for example. Here the type of the miterbends is the same, i.e., both are E-bends or H-bends. The electric field at the position (x, y') on the k-th miterbend reflector is described by the following equation:

$$\boldsymbol{e}_{tot}(x,y',L_k) = \sum_{\sigma=0}^{N}\sqrt{p_\sigma}\exp\{j(\phi_\sigma - k_\sigma(y'\sin\theta + L_k)\} \boldsymbol{E}_{\perp\sigma}(x, y'\cos\theta). \tag{6}$$

According to the array of Peltier devices, the reflector surface can be divided into $M \times M$ rectangular sections with the center coordinate of (x_i, y'_j) and the lengths of Δx and $\Delta y'$ in x-, y'-direction, respectively. The absorbed power in each segment is proportional to

$$|\boldsymbol{e}^D_{tot}(x_i, y'_j, L_k)|^2 \equiv \int_{x_i-\Delta x/2}^{x_i+\Delta x/2} dx \int_{y'_j-\Delta y'/2}^{y'_j+\Delta y'/2} dy' \, |\boldsymbol{e}_{tot}(x, y', L_k)|^2, \tag{7}$$

$$x_i = \Delta x\left(i - \frac{M-1}{2}\right) \quad (i = 0, \cdots, M-1), \tag{8}$$

$$y'_j = \Delta y'\left(j - \frac{M-1}{2}\right) \quad (j = 0, \cdots, M-1), \tag{9}$$

and the integrand can be expressed as

$$|\boldsymbol{e}_{tot}(x, y', L_k)|^2 = \sum_{\sigma=0}^{N} p_\sigma |\boldsymbol{E}_{\perp\sigma}(x, y'\cos\theta)|^2 + \sum_{\substack{\sigma=0 \\ \sigma\neq\tau}}^{N}\sum_{\tau=0}^{N}\sqrt{p_\sigma p_\tau}\cos(\phi_{\sigma\tau} - k_{\sigma\tau}(y'\sin\theta + L_k)) \times \boldsymbol{E}_{\perp\sigma}(x, y'\cos\theta)\cdot\boldsymbol{E}^*_{\perp\tau}(x, y'\cos\theta), \tag{10}$$

where $\phi_{\sigma\tau} = \phi_\sigma - \phi_\tau$, $k_{\sigma\tau} = k_\sigma - k_\tau$, and p_σ, ϕ_σ, and k_σ are the power, the initial phase, and the wave-number of the propagating mode σ, respectively. $\boldsymbol{E}_{\perp\sigma}$ corresponds to the electric field given by the equations (1)-(4). The evaluation function W_{tot} for determining mode content is defined by the segment and position summation of the square value of the difference between the measured O and theoretical T functions at each miterbend position,

$$W_{tot}(p_\sigma, \phi_\sigma) = \sum_{k=0}^{n-1} W(L_k), \tag{11}$$

where

$$W(L_k) = \sum_{i=0}^{M-1}\sum_{j=0}^{M-1}\{O(x_i, y'_j, L_k) - T(x_i, y'_j, L_k)\}^2, \tag{12}$$

$$T(x_i, y_j, L_k) = \frac{|\boldsymbol{e}^D_{tot}(x_i, y'_j, L_k)|^2}{|\boldsymbol{e}^D_{tot}|^2_{MAX}}. \tag{13}$$

The mode with $\sigma = 0$ is assumed to be the LP_{01} fundamental mode with the initial phase $\phi_0 = 0$, and $\sum_{\sigma=0}^{N} p_\sigma = 1$. The sets of p_σ and ϕ_σ which minimize the value of $W_{tot}(p_\sigma, \phi_\sigma)$ give the ratio of mode content and the initial phase of each mode σ. One of the techniques to minimize W_{out} is proposed in Ref. [1] in detail.

1) K. Ohkubo, et al., Fusion Science and Technology **62** (2012) 389.

2) T. Shimozuma, et al., J Infrared Milli Terahz Waves **37** (2016) 87–99.