§10. Performance Measurement of Arbitrary Waveform and Arbitrary Power Factor Matrix Converter

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Quaternion, four-dimensional hyper-complex number, is good at dealing with description of three-dimensional rotation as seen in three-dimensional game graphics programming theory[1)]. Now, instead of transforming three-phase AC to two dimension, to represent three-phase AC in three dimension, let's introduce quaternion, which is extended from a complex number[2)].

$$q = a + iv_x + jv_y + kv_z = a + v \tag{1}$$

$$\begin{cases} i^2 = j^2 = k^2 = -1 \\ ij = -ji = k, jk = -kj = i, ki = -ik = j \end{cases} \tag{2}$$

Let's assign three-phase AC phase (line-to-neutral) voltages to vector part of quaternion.

$$\begin{aligned} v &= \sqrt{2}V\{+i\cos\omega t + j\cos(\omega t - 2\pi/3) \\ &\quad + k\cos(\omega t - 4\pi/3) = \epsilon^{+\hat{n}\omega t/2}\sqrt{2}VV_0\epsilon^{-\hat{n}\omega t/2} \end{aligned} \tag{3}$$

$$\begin{cases} \hat{n} = (+i + j + k)/\sqrt{3} \\ V_0 = \{+i + j\cos(-2\pi/3) + k\cos(-4\pi/3)\} \end{cases} \tag{4}$$

Namely, it represents that the initial three-phase (positive phase) AC voltage vector V_0 rotates in counter clockwise with an axis of unit vector $\hat{n}$.

In order to manipulate the three-phase AC similarly as complex vector (phasor) of single-phase AC and symmetrical coordinate method of three-phase AC, let's introduce biquaternion.

$$h^2 = -1, ih = hi, jh = hj, kh = hk \tag{5}$$

Namely, we add a complex number h to the quaternion, which is exchangible with quaternion and independent from quaternion. And we assign complex vector representation and the exponential representation of each phase to the complex number h.

$$\begin{aligned} v &= \sqrt{2}V\{+i\epsilon^{h(\omega t+\phi)} + j\epsilon^{h(\omega t+\phi-2\pi/3)} + k\epsilon^{h(\omega t+\phi-4\pi/3)}\} \\ &= \epsilon^{+\hat{n}\omega t/2}\sqrt{2}VV_\phi\epsilon^{-\hat{n}\omega t/2} \\ &= \{R + p(L-M)\}\epsilon^{+\hat{n}\omega t/2}\sqrt{2}II_0\epsilon^{-\hat{n}\omega t/2} \end{aligned} \tag{6}$$

$$V_\phi = \{+i\epsilon^{h\phi} + j\epsilon^{h(\phi-2\pi/3)} + k\epsilon^{h(\phi-4\pi/3)}\} \tag{7}$$

Namely, let's combine the effective value and initial biquaternion into biquaternion, and we can obtain the same representation as the complex vector representation of single-phase AC. And we have only to use unit pure quaternion $\hat{n}$ instead of imaginary number j.

In complex number equation $v = Cu$, we can calculate the transformation $C = v/u$ by dividing the number v by the number u. Similarly in switching equation for matrix converter, we can express the three-phase input and output voltages by using quaternion.

$$\varepsilon^{+n\omega_o t}V_o = Q\varepsilon^{+n\omega_i t}V_i \tag{8}$$

In this case, the switching quaternion Q is calculated as follows.

$$\begin{aligned} Q &= \varepsilon^{+n\omega_o t}V_o(V_i)^{-1}\varepsilon^{-n\omega_i t} \\ &= r\epsilon^{+n(\omega_o-\omega_i)t} \end{aligned} \tag{9}$$

Here, r is the voltage transfer ratio. The above quaternion equation is re-expressed by vector equation with switching matrix.

$$\varepsilon^{+n\omega_o t}V_o = r\varepsilon^{+n(\omega_o-\omega_i)t}\varepsilon^{+n\omega_i t}V_i \tag{10}$$

$$\begin{aligned} &V_0\begin{bmatrix}\cos(\omega_0 t - 0\pi/3)\\ \cos(\omega_0 t - 2\pi/3)\\ \cos(\omega_0 t - 4\pi/3)\end{bmatrix} \\ &= \begin{bmatrix} r\cos\omega_m t & -r/\sqrt{3}\sin\omega_m t & +r/\sqrt{3}\sin\omega_m t \\ +r/\sqrt{3}\sin\omega_m t & r\cos\omega_m t & -r/\sqrt{3}\sin\omega_m t \\ -r/\sqrt{3}\sin\omega_m t & +r/\sqrt{3}\sin\omega_m t & r\cos\omega_m t \end{bmatrix} \\ &V_i\begin{bmatrix}\cos(\omega_i t - 0\pi/3)\\ \cos(\omega_i t - 2\pi/3)\\ \cos(\omega_i t - 4\pi/3)\end{bmatrix} \end{aligned} \tag{11}$$

$$\omega_m = \omega_o - \omega_i \tag{12}$$

For all components to be larger than 0 and smaller than 1, we have only to multiply 1/2 and add 1/2. But that's too bad, the summation of three elements in each row is not constant and cannot be made unity by any means as far as we consider in phase voltage.

By considering the line-to-line voltage, we can deduce original Venturini method.

$$\begin{aligned} &V_0\begin{bmatrix}\cos(\omega_0 t - 0\pi/3)\\ \cos(\omega_0 t - 2\pi/3)\\ \cos(\omega_0 t - 4\pi/3)\end{bmatrix} = \begin{bmatrix} 0 & +1 & -1 \\ -1 & 0 & +1 \\ +1 & -1 & 0 \end{bmatrix} \cdot [\text{rhs of eq.(11)}] \\ &= \frac{2r}{\sqrt{3}}\begin{bmatrix} \sin\omega_m t & \sin(\omega_m t + 2\pi/3) & \sin(\omega_m t + 4\pi/3) \\ \sin(\omega_m t - 2\pi/3) & \sin\omega_m t & \sin(\omega_m t + 2\pi/3) \\ \sin(\omega_m t - 4\pi/3) & \sin(\omega_m t - 2\pi/3) & \sin\omega_m t \end{bmatrix} \\ &V_i\begin{bmatrix}\cos(\omega_i t - 0\pi/3)\\ \cos(\omega_i t - 2\pi/3)\\ \cos(\omega_i t - 4\pi/3)\end{bmatrix} \end{aligned} \tag{13}$$

Concerning improved Venturini method, third higher harmonics of desired output phase voltage and input phase voltage can be added, since the harmonics constitute zero space about the transformation from phase voltage to line-to-line voltage. The quaternion characteristics will be utilized to analyze matrix converter based on improved Venturini method and space vector method in more detail.

1) E. Lengyel, "Mathematics for 3D Game Programming & Computer Graphics" 2nd ed., *Charles River Media, Inc.*, 2004.

2) J. H. Conway, D. Smith, "On Quaternions and Octonions: Their Geometry, Arithmetic, and Symmetry", *A. K. Perters, Ltd.*, 2003.