## §9. Radially Local Approximation of the Drift Kinetic Equation

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Under the new radially local approximation made in the present work [1], the guiding center equations are written as

$$\begin{aligned}\frac{d\mathbf{X}}{dt} &= \mathbf{V}_{\rm gc}^{(\rm rl)} \equiv U\mathbf{b} + (\mathbf{V}_{\rm gc}^{(\rm rl)})_\perp, \\ \frac{dU}{dt} &= -\frac{\mu}{m}\mathbf{b}\cdot\nabla B + U\mathbf{b}\cdot\nabla\mathbf{b}\cdot\mathbf{V}_{\rm gc}^{(\rm rl)} \\ \frac{d\mu}{dt} &= -\frac{1}{B}(\mathbf{V}_{\rm gc}^{(\rm rl)})_\perp\cdot(mU^2\mathbf{b}\cdot\nabla\mathbf{b}+\mu\nabla B), \quad (1)\end{aligned}$$

where $\mathbf{X}$, $U$, and $\mu$ are the guiding center position, parallel velocity, and magnetic moment, respectively. The component $(\mathbf{V}_{\rm gc}^{(\rm rl)})_\perp$ of the guiding center velocity $(\mathbf{V}_{\rm gc}^{(\rm rl)})$ perpendicular to the magnetic field $\mathbf{B} = B\mathbf{b}$ is defined by $(\mathbf{V}_{\rm gc}^{(\rm rl)})_\perp = \alpha(\Lambda)(c/eB)(\mathbf{b}\times\nabla s)\left[(\nabla s/|\nabla s|^2)\cdot(mU^2\mathbf{b}\cdot\nabla\mathbf{b}+\mu\nabla B)+e(d\Phi/ds)\right]$, where $s$ is an arbitrary label of the flux surface, $\Phi = \Phi(s)$ represents the electrostatic potential, and the factor $\alpha(\Lambda)$ is introduced to satisfy the condition $(\mathbf{V}_{\rm gc}^{(\rm rl)})_\perp(\mu = 0) = 0$ in order to derive appropriate balance equations of particles, energy, and parallel momentum by removing improper sources and/or sinks at the boundary $\mu = 0$ in the velocity-space integral domain.

The ratio of the magnetic moment $\mu$ to the kinetic energy $W = mU^2/2 + \mu B$ is used to define the dimensionless parameter, $\Lambda \equiv \mu B_{\rm max}/W$, where $B_{\rm max}$ is the maximum value of $B$ on the flux surface. Then, we assume that $\lim_{\Lambda\to+0}\alpha(\Lambda) = 0$ while $\alpha(\Lambda) = 1$ except for an interval, $0 \le \Lambda < \Lambda_0$, where $\Lambda_0(\ll 1)$ is a small positive constant value. For example, $\alpha(\Lambda)$ is defined by $\alpha(\Lambda) = \sin(\pi\Lambda/2\Lambda_0)$ for $\Lambda < \Lambda_0$, and 1 for $\Lambda \ge \Lambda_0$. We should note that influences of the magnetic and $\mathbf{E}\times\mathbf{B}$ drift motions are significant mainly for precession drift orbits of trapped particles, and that particles in the region, $\Lambda < \Lambda_0$, are passing ones whose orbits almost coincide with field lines. Therefore, even if the functional form of $\alpha(\Lambda)$ and the value of $\Lambda_0$ are changed, the artificial reduction factor $\alpha(\Lambda)$ for $\Lambda < \Lambda_0$ is expected to cause little change in resultant passing particles' orbits except that the limiting condition, $\lim_{\Lambda\to+0}(\mathbf{V}_{\rm gc}^{(\rm rl)})_\perp = 0$, is rigorously satisfied.

Note that $\mathbf{V}_{\rm gc}^{(\rm rl)}$ has no radial component: $\mathbf{V}_{\rm gc}^{(\rm rl)}\cdot\nabla s = 0$. In Eq. (1), the magnetic moment $\mu$ is allowed to vary in time such that conservation of the kinetic energy of the particle $W = mU^2/2 + \mu B$,

$$\frac{dW}{dt} = mU\frac{dU}{dt} + B\frac{d\mu}{dt} + \mu\mathbf{V}_{\rm gc}^{(\rm rl)}\cdot\nabla B = 0, \qquad (2)$$

is satisfied. We hereafter employ $(s,\theta,\zeta,W,U,\xi)$ as phase-space coordinates. Here, $(s,\theta,\zeta)$ are the flux surface coordinates for which the Jacobian is written as $\sqrt{g} = [\nabla s\cdot(\nabla\theta\times\nabla\zeta)]^{-1}$. Using $\mathbf{V}_{\rm gc}^{(\rm rl)}$, $dU/dt$, and $dW/dt = 0$ given by Eqs. (1) and (2), the drift kinetic equation for the first-order distribution function $f(s,\theta,\zeta,W,U)$ in the stationary state is written as

$$\frac{1}{\sqrt{g}}\left[\frac{\partial}{\partial\theta}(\sqrt{g}f\mathbf{V}_{\rm gc}^{(\rm rl)}\cdot\nabla\theta) + \frac{\partial}{\partial\zeta}(\sqrt{g}f\mathbf{V}_{\rm gc}^{(\rm rl)}\cdot\nabla\zeta)\right] + \frac{\partial}{\partial U}\left(f\frac{dU}{dt}\right)$$
$$= \frac{F_0}{T_0}\left\{V_{\rm gc}^s\left[X_1 + X_2\left(\frac{W}{T_0}-\frac{5}{2}\right)\right] + \frac{eUB}{\langle B^2\rangle^{1/2}}X_E\right\} + C^L(f), \quad (3)$$

where the thermodynamic forces $X_1$, $X_2$, and $X_E$ are defined by $X_1 = -(1/n_0)(\partial p_0/\partial s) - e(\partial\Phi/\partial s)$, $X_2 = -\partial T_0/\partial s$, and $X_E = \langle BE_\parallel\rangle/\langle B^2\rangle^{1/2}$, respectively, $C^L(f)$ represents the linearized collision operator, $F_0$ is the zeroth-order distribution function which takes the local Maxwellian form, and the radial component of the guiding center drift velocity $V_{\rm gc}^s$ is given by $V_{\rm gc}^s = (c/eB^2)[\nabla s\cdot(\mathbf{b}\times\nabla B)]\left(\frac{1}{2}mU^2 + W\right)$. Note that the radial coordinate $s$ and the kinetic energy $W$ enter $f(s,\theta,\zeta,W,U)$ as constant parameters.

The new drift kinetic equation (3), which includes both $\mathbf{E}\times\mathbf{B}$ and tangential magnetic drift terms, has such a favorable property for numerical simulation that any additional terms for particle and energy sources are unnecessary for obtaining stationary solutions under the radially local approximation. It is also shown that the solutions of Eq. (3) satisfy the intrinsic ambipolarity condition for neoclassical particle fluxes in the presence of quasisymmetry of the magnetic field strength. In [1], another radially local drift kinetic equation different from Eq. (3) is presented, from which the positive definiteness of entropy production due to neoclassical transport and Onsager symmetry of neoclassical transport coefficients are derived while it sacrifices the ambipolarity condition for neoclassical particle fluxes in axisymmetric and quasisymmetric systems.

1) H. Sugama, S. Matsuoka, S. Satake, R. Kanno, Phys. Plasmas **23**, 042502 (2016).