§15. Effects of Collisions on Conservation Laws in Gyrokinetic Field Theory

Sugama, H., Watanabe, T.-H. (Nagoya Univ.),
Nunami, M.

In the extended gyrokinetic field theory [1,2], all governing equations for the gyrocenter motion and the electromagnetic fields in the collisionless gyrokinetic system are derived from the variational principle, $\delta\mathcal{I} = 0$, where $\mathcal{I} \equiv \int_{t_1}^{t_2} L dt$ is the action integral and L is the Lagrangian which includes not only the gyrocenter coordinates and turbulent electromagnetic fields but also the constraint on the background magnetic fields. Here, the background magnetic field configuration is assumed to be axisymmetric. Then, conservation laws of energy and toroidal angular momentum for the collisionless system are naturally derived from Noether's theorem. In the present study [3], we investigate how the collision and external source terms added into the gyrokinetic equations influence the conservation laws derived from the gyrokinetic field theory.

In the presence of collisions, the gyrocenter distribution function $F_a(\mathbf{Z}, t)$ for species a is governed by the gyrokinetic Boltzmann equation,

$$\begin{aligned}&\left(\frac{\partial}{\partial t} + \frac{d\mathbf{Z}_a}{dt} \cdot \frac{\partial}{\partial \mathbf{Z}}\right) F_a(\mathbf{Z}, t) \\ &\quad = \sum_a \langle C^g_{ab}[F_a, F_b](\mathbf{Z}, t)\rangle_\xi + \mathcal{S}_a(\mathbf{Z}, t), \end{aligned} \tag{1}$$

where $d\mathbf{Z}_a/dt$ is the time derivative of the gyrocenter coordinates $\mathbf{Z}_a = (\mathbf{X}_a, U_a, \mu_a, \xi_a)$ along the collisionless orbit, $\langle C^g_{ab}[F_a, F_b](\mathbf{Z}, t)\rangle_\xi$ represents the gyrophase-averaged collision term, and $\mathcal{S}_a(\mathbf{Z}, t)$ denotes other parts including external particle, momentum, and/or energy sources if any. Here, $F_a(\mathbf{Z}, t)$ and $\mathcal{S}_a(\mathbf{Z}, t)$ are both regarded as independent of the gyrophase ξ.

Associating the collisional system with a corresponding collisionless system at a given time such that the two systems have the same distribution functions and electromagnetic fields instantaneously, it is shown how the collisionless conservation laws derived from Noether's theorem are modified by the collision term and the external source term. Also, a novel approximate gyrokinetic collision operator is derived in [3], where desirable conservative (or divergence) forms of classical particle, energy, and toroidal momentum fluxes can be derived from the gyrokinetic velocity-space integrals using the collision operator.

Integrating the gyrokinetic Boltzmann equation, Eq. (1), with respect to the gyrocenter velocity-space coordinates (U, μ, ξ), we obtain the evolution equation for the gyrocenter density $n_a^{(\mathrm{gc})} = \int dU \int d\mu \int d\xi\, D_a F_a$ which is written as

$$\frac{\partial n_a^{(\mathrm{gc})}}{\partial t} + \nabla \cdot \left(\boldsymbol{\Gamma}_a^{(\mathrm{gc})} + \boldsymbol{\Gamma}_a^{\mathrm{C}}\right) = \int dU \int d\mu \int d\xi\, D_a \mathcal{S}_a, \tag{2}$$

where $\boldsymbol{\Gamma}_a^{(\mathrm{gc})} = \int dU \int d\mu \int d\xi\, D_a F_a d\mathbf{X}_a/dt$. Here, $\boldsymbol{\Gamma}_a^{\mathrm{C}}$, which is defined in terms of the collision operator, represents the classical particle flux while the neoclassical and turbulent particle fluxes are included in $\boldsymbol{\Gamma}_a^{(\mathrm{gc})}$. The right-hand side of Eq. (2) gives the particle source.

Using Noether's theorem modified for the collisional system with the external source, the equation for the flux-surface-averaged energy density $\langle\mathcal{E}\rangle$ is derived as

$$\begin{aligned}&\frac{\partial}{\partial t}\left(V' \langle\mathcal{E}\rangle\right) + \frac{\partial}{\partial s}\left(V' \langle(\mathbf{Q} - \mathcal{E}\mathbf{u}_s) \cdot \nabla s\rangle\right) \\ &\quad = V' \sum_a \left\langle \int dU \int d\mu \int d\xi\, D_a \mathcal{S}_a H_a \right\rangle, \end{aligned} \tag{3}$$

where $\langle\cdots\rangle$ stands for the flux-surface average, $\mathbf{u}_s \cdot \nabla s$ represents the radial velocity of the flux surface labeled by s, $V' = dV/ds$ is the radial derivative of the volume V enclosed by the flux surface, and H_a denotes the gyrocenter Hamiltonian. Note that $\mathcal{E}$ contains both kinetic and field energies and that the classical energy flux $\mathbf{Q}^{\mathrm{C}}$, which is defined in terms of the collision operator, is included in the energy flux $\mathbf{Q}$. The toroidal angular momentum balance equation for the collsional system with the external source can also be derived from the modified Noether's theorem.

The ensemble-averaged transport equations of particles, energy, and toroidal momentum given in the present work [3] are shown to include classical, neoclassical, and turbulent transport fluxes which agree with those derived from conventional recursive formulations with WKB representation.

1) H. Sugama, Phys. Plasmas **7**, 466 (2000).
2) H. Sugama, T.-H. Watanabe, M. Nunami, Phys. Plasmas **21**, 012515 (2014).
3) H. Sugama, T.-H. Watanabe, M. Nunami, Phys. Plasmas **22**, 082306 (2015).