# §19. A Structural Bifurcation of Transport in Toroidal Plasmas†

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The structure formation mechanism, which is caused by the nonlinear link between the radial electric field, pressure gradient and turbulent fluctuations, is investigated. The suppression of turbulence by the shear and curvature of inhomogeneous radial electric field is taken into account. The case of L-mode, where zonal flows are not exited, is studied. The one-dimensional structure is investigated, and the stationary state under the given heat flux (constant in space and time) is analyzed. It is shown that the effect of electric field curvature causes the structural bifurcation from the state of constant gradient to the state with spatially-corrugated gradient [1]. The relation between the period and amplitude of the corrugation is obtained. The electric field shear alone does not lead to such a structural bifurcation.

The motivation of the present analysis is to show the nature of inhomogeneity, that is, the gradient is not necessarily be a smooth function in radius even though the given heat flux is constant in space. The structure formation mechanisms, in which the combined effects of electric field inhomogeneity, turbulence suppression and energy transport are taken into account, are studied. The first assumption is that the simplest model for L-mode plasmas is analyzed for the demonstration of ubiquitous corrugation in gradient. The turbulence-driven transport is assumed induced by microscopic fluctuations. It is also assumed that the radial electric field is composed of the mean (dc) radial electric field.

The stationary state solution for the turbulence intensity is given as

$$I = \left(1 + \rho_i^2 \left(\frac{1}{V_d B}\frac{\mathrm{d}E_r}{\mathrm{d}r}\right)^2 - \rho_i^2 \left(\frac{1}{V_d B}E_r\right)\left(\frac{1}{V_d B}\frac{\mathrm{d}^2 E_r}{\mathrm{d}r^2}\right)\right)^{-1} I_0 \qquad (1)$$

where $I$ is the normalized density of fluctuation energy of interests [2]. We introduce a normalized temperature gradient, G = $-(a/T)\mathrm{d}T/\mathrm{d}r$, and the radial electric field is proportional to the gradient of temperature and is simply rewritten as $E_r/BV_d = \alpha G$, where $\alpha$ is a constant of the order unity. Thus, the nonlinear interaction of gradient on the turbulence intensity appears according to Eq.(1). The turbulence-driven thermal conductivity $\chi_{\mathrm{turb}}$ is modified as

$$\chi_{\mathrm{turb}} = \chi_0 \left(1 - \alpha^2 \rho_i^2 \left\{G G'' - (G')^2\right\}\right)^{-1} \qquad (2)$$

where $\chi_0$ is the thermal conductivity in the absence of corrugations of electric field. Thus the transport equation $-\chi_{\mathrm{turb}}\, n \nabla T = Q_r$ is rewritten as

$$G^2 \left(1 - \alpha^2 \rho_i^2 \left\{G G'' - (G')^2\right\}\right)^{-1} = G_0^2 \qquad (3)$$

where $G_0^2 = Q_r \left(nT\left(\rho_i T/aeB\right)\right)^{-1}$ is the gradient where the corrugation is not taken into account.

For the transparency of the argument, we introduce the normalized variables $g = G/G_0$ and $x = (r - r_0)/\alpha G_0 \rho_i$, and Eq.(3) takes the form

$$g^2 \left(1 - g g'' + (g')^2\right)^{-1} = 1 \qquad (4)$$

The solution of the constant gradient is obtained from Eq.(4) as $g = 1$. The oscillatory nonlineat solution is also obtained from Eq.(4). Equation (4) is integrated as

$$(g')^2 = F(g) \equiv -1 - 2g^2 \ln g + Cg^2 , \qquad (5)$$

where $C_0$ is a constant of integral. The function $F(g)$ is shown in Fig.1. It is positive in the bounded region $g_1 < g < g_2$, where $F(g_1) = F(g_2) = 0$ holds, for $C_0 \geq 1$. An oscillatory solution of $g(x)$ that satisfies $g_1 \leq g(x) \leq g_2$ is allowed if $C_0 \geq 1$. That is, the gradient is no longer smooth but has self-generated corrugations. The case of $C_0 = 1$ corresponds to the solution of constant gradient, $g(x) = 1$.

Under the condition of given heat flux (constant in space), we found that the turbulence suppression mechanism by curvature of electric field causes the structural bifurcation from the state of constant gradient to the state with corrugated gradient. This finding stimulates the search of small-scale corrugation of 'mean' plasma parameters [3].

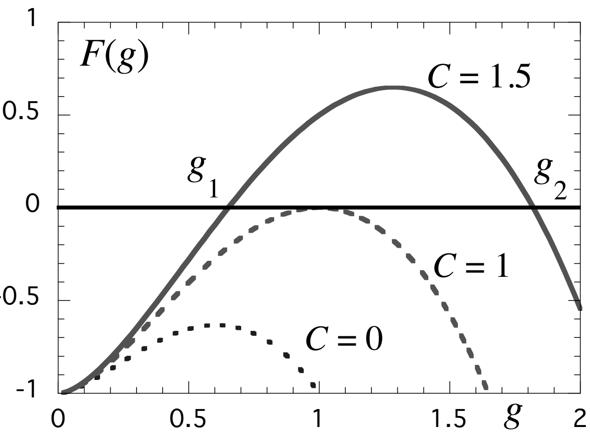


**Fig.1**: Function $(g')^2 = F(g)$ for various values of the constant of integral $C$.

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