§21. A Reduced Fluid Model and the Law of Momentum Conservation

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Coupling of plasma turbulence with flows is an important subject for understanding turbulent transport in magnetized plasmas. Zonal flows, which are meso-scale poloidal flows driven by micro-scale turbulent fluctuations, have been intensively studied [1]. Generation mechanisms of toroidal intrinsic rotation have been also investigated [2]. Toward understanding of L-H transition, an interaction of micro-scale turbulence with macro-scale poloidal flow has been studied theoretically [3], and the turbulence simulation has been reported based on Hasegawa-Wakatani type reduced model [4]. Study on dynamical response of turbulence with an externally driven flows has also started [5]. For the study of formation of flows, it is essentially important to use a model which guarantees the law of momentum conservation. Relation of reduced models with the conservation law has been discussed [6, 7]. In order to investigate the coupling of the flow and turbulence, the relation between vorticity and the velocity field should be elucidated. In this study, a reduced model which satisfies the momentum conservation was derived, and a turbulence simulation code was developed by using its model.

A reduced fluid model which guarantees the conservation law is derived, which is an extension of Hasegawa-Wakatani model, and is compared with the Numerical Linear Device (NLD) code [8, 9]. In the following, we consider cylindrical plasmas. The units of time, space and velocity are chosen as ion cyclotron frequency, ion Larmor radius, and the sound velocity, respectively. In this report, the derivation of a vorticity equation is focused on. The time evolution of the momentum density is governed as

$$\frac{\partial}{\partial t} n\boldsymbol{v} + \nabla \cdot (n\boldsymbol{v}\boldsymbol{v}) = -\nabla n + \boldsymbol{J} \times \boldsymbol{b} - \nu n\boldsymbol{v} + \boldsymbol{S}_p. \quad (1)$$

Here, $\boldsymbol{J}$ is the current density, $\boldsymbol{b}$ is the unit vector in the magnetic field direction, ν is the collisional frequency, and $\boldsymbol{S}_p$ is the momentum source. The perpendicular current density can be obtained by taking the cross product of $\boldsymbol{b}$ to Eq. (1). The parallel current density is assumed to be determined by the electron velocity as $J_\parallel = nV$. Then, the vorticity equation is obtained by using the charge neutrality condition $\nabla \cdot \boldsymbol{J} = 0$ as

$$\frac{\partial}{\partial t}\Omega + \nabla \cdot \left[\nabla \cdot \Pi\right] = -\nabla_\parallel (nV) - \nu\Omega + S_U. \quad (2)$$

Here the velocity field is assumed to be $E \times B$ drift velocity, $\boldsymbol{v} = \nabla_\perp \phi \times \boldsymbol{b}$. The vorticity is obtained as $\Omega = \nabla_\perp \cdot (n \nabla_\perp \phi)$, the stress tensor is $\Pi = n\boldsymbol{v}\boldsymbol{v}$, and the relation of the vorticity source with the momentum source is obtained, which is expressed as $\nabla \cdot (\boldsymbol{S}_p \times \boldsymbol{b})$. It is noted that the stress tensor includes the 3rd order nonlinearity such as $\tilde{n}\tilde{v}_r\tilde{v}_\theta$. In order to elucidate the relationship between the obtained model and the previous one, Eq. (2) is rewritten by the evolution of $\nabla_\perp^2 \phi$ as

$$\begin{aligned}
&\frac{d}{dt}\nabla_\perp^2 \phi + \nabla N \cdot \frac{d}{dt}\nabla_\perp \phi = -\nabla_\parallel V - V \nabla_\parallel N \\
&- \left(\nabla_\perp^2 \phi + \nabla_\perp \phi \cdot \nabla N + \nabla_\perp \phi \cdot \nabla\right)\left(\frac{\partial N}{\partial t} + [\phi, N]\right) \\
&-\nu\left(\nabla N \cdot \nabla_\perp \phi + \nabla_\perp^2 \phi\right) + e^{-N} S_U, \qquad (3)
\end{aligned}$$

where $N = \ln(n/n_0)$. The extension points are 1) the 3rd order nonlinear term, which is related to the convective derivative of the second term in the LHS, and 2) the term related to the density dissipation and the source, which is the third term in the RHS. In the case of the flux driven simulation, the density source term is necessary. In such cases, the density correction term is important to be considered, otherwise the perpendicular velocity which stems from the density source is generated. The turbulence simulation code was developed by using the obtained model. The effect of 3rd order nonlinear term to the turbulence and the flow formation and the dynamical response of turbulence to an externally driven flow will be reported in near future.

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