§13. Statistical Laws of Strong Fluctuations and Cascade Flux in NS and MHD Turbulent Mixing Studied by Massive Parallel Computation

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Fluctuations of velocity, scalar, and magnetic fields in the NS and MHD turbulence become stronger with decrease of spacial scale. The moments of scalar increment with the separation distance r are believed to obey a power law $S_n(r) = \langle[\theta(x+r) - \theta(x)]^n\rangle \propto r^{\zeta_n}$ and the scaling exponent ζ_n to be universal. However, recent studies for the passive scalar suggest that the universality of the scaling exponent of the passive scalar in turbulence is not as robust or universal as in the case of the velocity.

We have conducted very large scale direct numerical simulations of two passive scalars which was convected by the same turbulent velocity and excited by two different ways. Scalar θ is excited by the Gaussian random source which is white in time and applied at low wavenumbers and scalar q by the uniform mean scalar gradient $d\bar{Q}/dz = \Gamma$. We found the remarkable facts that the local scaling exponents $\zeta_n^\alpha(r) = d\log S_n^\alpha(r)/d\log r$ ($\alpha = \theta, q$) are not universal and $\zeta_n^\theta(r)$ has the logarithmic correction as $\zeta_n^\theta(r) = \xi_n^\theta + \beta_n \log(r/r_*)$ [1].

To explore the reason for the difference in the local scaling exponents, we have made the large scale numerical simulations for two passive scalars

$$\partial\theta/\partial t + \boldsymbol{u}\cdot\nabla\theta = \kappa\nabla^2\theta + f_\theta,$$
$$\partial q/\partial t + \boldsymbol{u}\cdot\nabla q = \kappa\nabla^2 q + f_q.$$

for which $N = 2048^3, R_\lambda \approx 500$. But, the scalar injection methods were modified in such a way that f_θ obeyed the Ornstein-Uhlenbeck process instead of the white in time nature and the range of wavenumbers of f_q was restricted to the same low wavenumbers as that of the f_θ, that is,

$$\begin{aligned}
\langle f_\theta(\boldsymbol{k},t)f_\theta(-k,s)\rangle &= CH(3-k)\delta(t-s) \\
&\longrightarrow CH(3-k)e^{-u_0k|t-s|}, \\
\langle f_q(\boldsymbol{k},t)f_q(-k,s)\rangle &= \Gamma^2\langle u_3(\boldsymbol{k},t)u_3(-\boldsymbol{k},s)\rangle \\
&\longrightarrow \Gamma^2 H(3-k)\langle u_3(\boldsymbol{k},t)u_3(-\boldsymbol{k},s)\rangle.
\end{aligned}$$

where $H(t)$ is the Heaviside function. The results are shown in Fig.1. When the time correlation of the random scalar source is finite, the slope of $\zeta_n^\theta(r)$ becomes smaller. On the other hand, it is found from Fig.2 that the plateau regions of $\zeta_n^q(r)$ disappear. This finding means that the difference in the local scaling exponents arises from the difference in the wavenumber range of the excitation. For $f_q(\boldsymbol{k},t)$ without the filter, the scalar fluctuations are excited at all wavenumebrs directly by $u_3(\boldsymbol{k},t)$, so that the scalar fluctuations of q is in essence equivalent to those of u_3. On the other hand, it is not known why $\zeta_n^\theta(r)$ has the logarithmic correction, which certainly needs new theory of turbulence.

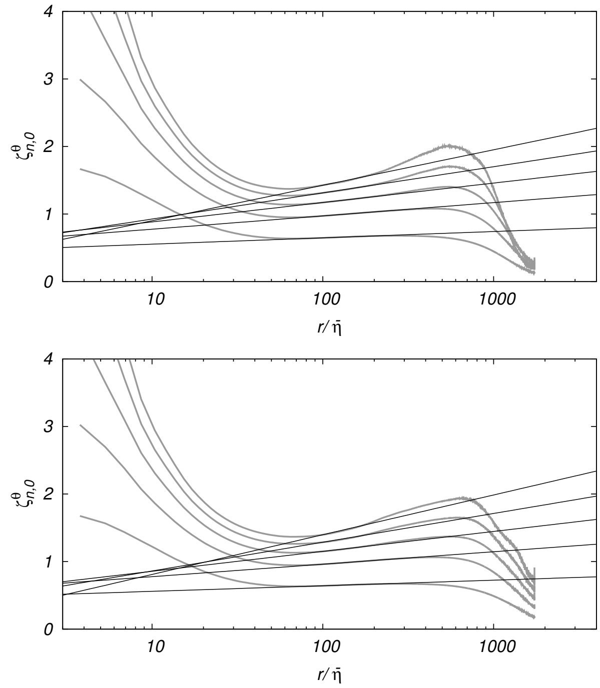

Fig. 1: Comparison of local scaling exponents $\zeta_{n0}^\theta(r)$ for the delta correlated injection (top) and for the finite time correlation (bottom) for $n = 2, 4, 6, 8, 10$.

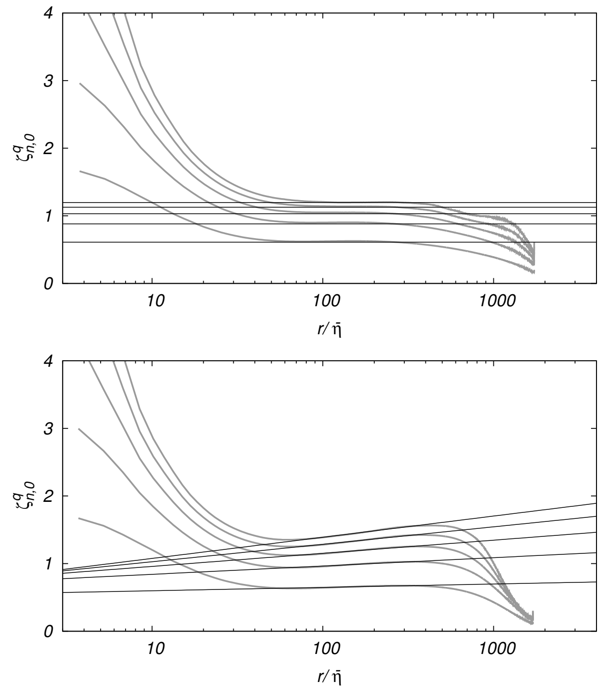

Fig. 2: Comparison of local scaling exponents $\zeta_{n0}^q(r)$ for unfiltered injection (top) and for filtered injection (bottom) for $n = 2, 4, 6, 8, 10$.

[1] Gotoh, T. and Watanabe, T.: Phys. Rev. Lett. **115**, (2015) 114502.