§16. Study on the Effect of the Hall Term on the Formation and Sustainment of the Current Sheet based on the Current Density Equation in the MHD Turbulence

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Vortical structures in turbulence are approximately divided into the two groups, i.e., the tube-like and sheet-like structures. In the MHD turbulence, the sheet-like structure known as current sheet is sustained and prevails. When the Hall effect is introduced into MHD, however, transition from sheet-like structure to tube-like structure occurs more frequently due to instability of the sheet structure (Miura and Araki 2014). It is considered that this is due to enhancement of the small-scale generation by the Hall term and formation and sustainment of sheet structure is affected. The purpose of this study is to elucidate the roles of the Hall effect on the current sheet formation.

The governing equation for the magnetic field is given by Eq. (1) and the convective term is represented by the upper convective derivative as

$$\frac{dB_i}{dt} = B_k \frac{\partial u_i}{\partial x_k} \quad (1)$$

Eq.(1) is similar to the vorticity equation and its solution is the contravariant vector in general. The governing equation for the current density becomes as Eq. (2).

$$\frac{dj_i}{dt} = \frac{1}{2} j_k \frac{\partial u_i}{\partial x_k} - \frac{1}{2} j_k \frac{\partial u_k}{\partial x_i} - \varepsilon_{ijk} S_{jl} \left(\frac{\partial B_k}{\partial x_l} + \frac{\partial B_l}{\partial x_k} \right) + B_k \frac{\partial \omega_i}{\partial x_k} \quad (2)$$

The first and second terms in the right-hand side of Eq.(2) are upper and lower convective derivatives, respectively. The solution of the lower convective derivative equation is the covariant vector. Summation of the first and second terms in Eq.(2) yields

$$\frac{1}{2} j_k \left(\frac{\partial u_i}{\partial x_k} - \frac{\partial u_k}{\partial x_i} \right) \quad (3).$$

and becomes the co-rotational convective derivative. Eq.(3) merely gives the rotation of the current density vector and no stretching of the current density vector is generated by this term. The third term in the right-hand side of Eq.(2) is primarily responsible for generation of the current density. Thereby, we conducted assessment on the correlation between the stretching of the magnetic field and generation of the fluid dissipation in the MHD turbulence.

We describe the Lorentz force in MHD in terms of the Maxwell stress as

$$(\boldsymbol{j} \times \boldsymbol{B})_i = \frac{\partial \tau_{ij}}{\partial x_j} \qquad \left(\tau_{ij} = B_i B_j - \frac{1}{2} \delta_{ij} |\boldsymbol{B}|^2 \right) \quad (4)$$

The governing equation for the Maxwell stress becomes analogous to the constitutive equation for the polymer stress in the viscoelastic fluid, i.e., the Oldroyd-B equation when the relaxation term for the magnetic field is added. In this study, we derive the approximate solution for the Maxwell tensor by expanding the solution with respect to the relaxation time.

When the magnetic field is highly stretched, the production term for the magnetic energy P_B becomes as

$$P_B = B_i B_j S_{ij} \approx 4\lambda S_{ik} S_{kj} S_{ij} \quad (5)$$

where λ denotes the relaxation time, whereas the production term for the dissipation rate becomes as

$$\frac{d}{dt}\left(\frac{1}{2} S_{ij} S_{ji} \right) = -S_{ik} S_{kj} S_{ji} \equiv P_\varepsilon \quad (6)$$

where S_{ij} denotes the strain-rate tensor.

Because the strain skewness $-S_{ik} S_{kj} S_{ji}$ is predominantly positive, we get $P_B < 0$ and $P_\varepsilon > 0$. Thus, when the magnetic field is highly stretched, the magnetic energy decreases, while the dissipation rate increases, i.e., the magnetic energy is converted to the fluid energy and it leads to enhancement of the dissipation.

Figure 1 (a) and 1 (b) show the probability density function (p.d.f.) for the P_B and P_ε terms, respectively. To obtain the p.d.f., the sampling of the data conditioned on the amplitudes of the $-S_{ik} S_{kj} S_{ji}$ term is applied. The lines plotted using the pink color shows the p.d.f. obtained when the $-S_{ik} S_{kj} S_{ji}$ term is smallest and the lines plotted using the purple shows the p.d.f. from the case in which the $-S_{ik} S_{kj} S_{ji}$ term is the largest. It is seen that as the amplitude of the $-S_{ik} S_{kj} S_{ji}$ term increases, the P_B term decreases, while the P_ε term increases. In accordance with the theoretical analysis, as the stretching of the magnetic field increases, the magnetic energy is indeed released and converted into the fluid energy and transferred to the dissipation of the fluid energy. We note that this result is similar to the result obtained in the viscoelastic turbulence diluted with the contravariant polymers, and shares a common feature of the turbulence governed by the contravariant elements.

Assessment on the effect of addition of the Hall term on this energy transfer process is left to the future work.

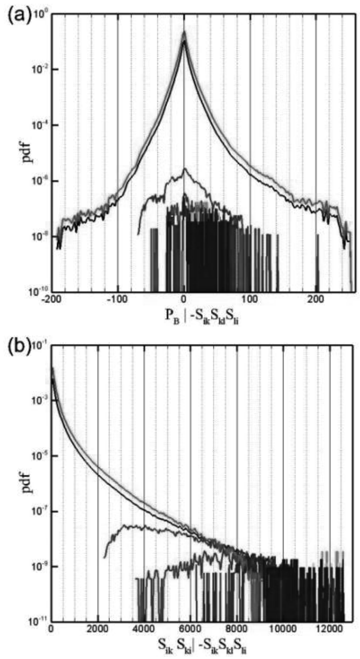

Figure 1 Distribution of the p.d.f. conditionally sampled on the amplitude of $-S_{ik} S_{kj} S_{ji}$; (a) the P_B term, (b) the P_ε term.