§17. MHD Simulation Inside a Sphere

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While a sphere plays a paradigmatic role for theoretical considerations, it is notoriously difficult to perform numerical computations in a sphere. The difficulty becomes apparent in the spherical polar coordinate system $\{r, \vartheta, \varphi\}$, with r radius, ϑ colatitude, and φ longitude. The numerical challenge resides near the coordinate singularities of the poles $\theta = 0, \pi$ and the origin $r = 0$, where grid points are highly concentrated. The dense grid points around a coordinate singularities drastically reduce the time step due to the Courant–Friedrichs–Lewy (CFL) condition when an explicit time integration scheme is adopted. Even when an implicit scheme is used, the unbalanced distribution of the grid spacings damages the computational efficiency because there is no physical reason, in general, to place a very fine grid mesh near a coordinate singularity.

For numerical simulations in a sphere, we have recently proposed an overset grid system, Yin–Yang–Zhong grid[1], which is an extension of the Yin–Yang grid.

The Yin–Yang–Zhong grid has three components; Yin, Yang, and Zhong (see Figure 1). The new component grid (Zhong), which is a set of cuboid blocks based on the Cartesian grid, is placed to cover the central part of the Yin–Yang grid. The three component grids cover the ball region with partial overlaps on their borders. The boundaries are sewed together by mutual interpolations, following the general overset grid methodology. Since the Yin–Yang–Zhong grid is a straightforward extension of the Yin–Yang grid, it is relatively easy to modify an existing Yin–Yang code into a Yin–Yang–Zhong code.

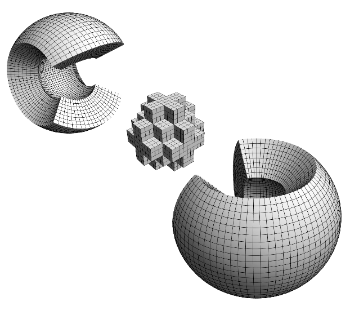

Fig. 1: A spherical overset grid system, Yin–Yang–Zhong.

When a magnetohydrodynamics (MHD) fluid with a magnetic field is placed in a vessel (without initial flow), the MHD system shifts spontaneously toward other states if the initial state is unstable. After a transition time, the system calms itself down to a quasi-equilibrium state. This process is called MHD relaxation[2]. Various plasma experiments show surprisingly good agreements with a relaxation theory proposed by Woltjer[3] and Taylor[4]. Although plasma instabilities—and consequently flows—play essential roles in the relaxation process in the Woltjer-Taylor theory, the flow velocity is assumed to be absent in the relaxed state in the theory.

We have performed an MHD simulation inside a sphere using the Yin–Yang–Zhong grid to investigate the MHD relaxation that has a flow in the relaxed state. Figure 2 shows streamlines in the relaxed state obtained by the simulation. The quasi-stationary, relaxed state has both the magnetic field and flow field with the same levels of energy. This is a solution beyond the Woltjer-Taylor theory. We are currently analyzing the dynamical process of the relaxation and interactions between the fields in the relaxed state.

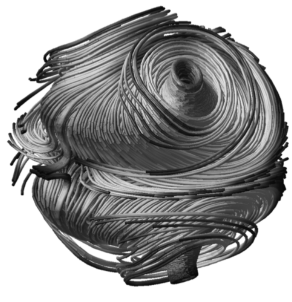

Fig. 2: Stream lines in an MHD relaxation in a sphere.

We have also performed an MHD simulation of thermal convection in a thin spherical shell layer with the Yin–Yang–Zhong grid. The layer is between two concentric spheres of radii $r = 0.9$ and $r = 1.0$, whose temperatures are kept hot and cold, respectively. A central gravity toward the center is assumed. The purpose of this simulation is to investigate the pattern formation of the MHD convection and the MHD dynamo effect by the flow. The MHD convection exhibits a roll-like pattern in the spherical shell. The Zhong grid component is critically important in this simulation because the dynamo-generated magnetic field diffuses into the inner sphere of $r \leq 0.9$, in which we solve the diffusion equation for the magnetic field on the Zhong grid.

1) H. Hayashi and A. Kageyama. Yin-Yang-Zhong grid: An overset grid system for a sphere. *Journal of Computational Physics*, 305:895-905, 2015.

2) S. Ortolani and Dalton D. Schnack. *Magnetohydrodynamics of plasma relaxation*. World Scientific, Singapore; River Edge, NJ, 1993.

3) L. Woltjer. A theorem on force-free magnetic fields. *Proc Natl Acad Sci U S A*, 44(6):489–491, 1958.

4) J. B. Taylor. Relaxation and magnetic reconnection in plasmas. *Reviews of Modern Physics*, 58(3):741, 1986.