§18. Development of a Multidimensional Higher-order Shock Capturing Scheme for Magnetohydrodynamics

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In space and astrophysical plasmas, MHD shocks and turbulence universally exist, and those interactions play important roles for various space and astrophysical phenomena. In fact, a recent high-resolution computer simulation for magnetic reconnection at very high Lundquist number reveals that small-scale Petschek-type shocks are dynamically generated in a turbulent current sheet[1].

As shock capturing methods for MHD which are able to suppress unphysical oscillations or wiggles around the MHD shocks and discontinuities, various approximate Riemann solvers have been developed so far. In particular, the Harten-Lax-van Leer-Discontinuities (HLLD) solver[2] is thought to be one of the promising solvers for MHD because of its high-resolution, efficiency, and robustness. On the other hand, higher-order methods are necessary to compute MHD turbulence so as not to damp small-scale vortices. Most of the higher-order methods adopted for MHD solvers are finite-volume approaches with higher-order nonlinear interpolations such as the so-called MUSCL, FV-WENO, and MP5. In general, it is not easy to extend the finite-volume method to higher-order (more than third) in multidimensions, while the higher-order finite-difference method may straightforwardly become multidimensional. Therefore, in this study, we construct a multidimensional higher-order shock capturing scheme for MHD by applying a particular finite-difference approach, named Weighted Compact Nonlinear Scheme (WCNS)[3].

The WCNS is constructed by combining a higher-order variable interpolation and a higher-order central finite-difference method. In the present report, the combinations of the weighted fifth-order Lagrange interpolation for the characteristic variables with the sixth-order finite-difference using the numerical fluxes at the midpoints (WCNS5-MD6), with the sixth-order finite-difference using the numerical and physical fluxes at the midpoints and nodes (WCNS5-MND6), and with the fourth-order finite-difference using the numerical and physical fluxes at the midpoints and nodes (WCNS5-MND4) are newly developed. In order to remove the numerical divergence of the magnetic field, the hyperbolic divergence cleaning method is applied. In addition, the third-order strong-stability-preserving (SPP) Runge-Kutta method is adopted as a time integration method.

Fig. 1 shows the results of the Orszag-Tang vortex problem. For comparison, the results computed by the second-order finite-volume MUSCL is also shown. The results clearly indicate that the WCNSs can sharply capture discontinuities in comparison with the MUSCL. Moreover, it is found from long-time simulations that small-scale vortices can be well resolved by all the present WCNSs, and the results does not clearly depend on the finite-difference methods at least in the present combinations. Note that the hyperbolic divergence cleaning method produces excessive numerical diffusivities in general. Therefore, a novel divergence-free WCNS with a genuinely multidimensional finite-difference method is under development.

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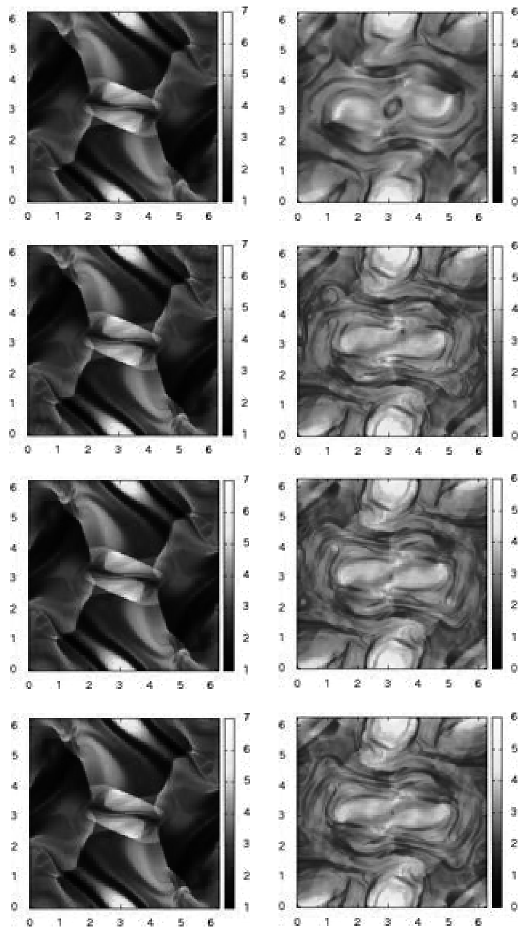

Fig.1: Results for the Orszag-Tang vortex problem at $t = \pi$ (left panels) and $t = 3\pi$ (right panels). From top to bottom: MUSCL, WCNS5-MD6, WCNS5-MND6, and WCNS5-MND4, respectively. The color shows the density distribution.

1) Shibayama, T., et al.: Phys. Plasmas **22** (2015) 100706.
2) Miyoshi, T., Kusano, K.: J. Comput. Phys. **208** (2005) 315.
3) Nonomura, T., Fujii, K.: Comput. Fluids **85** (2013) 8.