§20. Development of a Sub-grid-scale Model for LES of Magnetized Plasmas under a Two-fluid Effect

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for the purpose of carrying out Large Eddy Simulations (LESes) of Hall magnetohydrodynamic (MHD)/extended MHD simulations. It has been shown that the introduction of the Hall term to an MHD system brings about an extreme broadening of spatial and temporal spectra. The broadening can make a numerical simulation very difficult. In order to overcome the problem, we carry out LESes with a SGS model which represents a part of influences of the scales smaller than the grid width to the scales larger than the grid width.

We adopt the Smagorinsky-type SGS model for MHD channel turbulence developed by Hamba and Tsuchiya (PoP 2010). The SGS term which appears in the right-hand-side (RHS) of the equation of motions as well as the SGS terms which appears in the induction law are modeled as

$$\overline{\tau}_{ij} = -\nu_{SGS}\overline{S}_{ij},$$

$$\overline{E}_i^M = -\eta_{SGS}\overline{J}_i,$$

$$\nu_{SGS} = C_\nu \Delta^2 \left(\frac{1}{2}C_\nu \overline{S}_{ij}^2 + C_\eta \overline{J}_i^2\right)^{1/2},$$

$$\eta_{SGS} = C_\eta \Delta^2 \left(\frac{1}{2}C_\nu \overline{S}_{ij}^2 + C_\eta \overline{J}_i^2\right)^{1/2},$$

where τ_{ij}, S_{ij}, J_i are the SGS stress tensor, the rate of teh strain tensor and the current density, respectively. The two constants C_ν and C_η are called as the Smagorinsky constants. The symbol Δ is the geometric average of the grid width, and the overbar represents the variable is operated by the low-pass filter, the cut-off scale of which is given by Δ. By specifying the two Smagorinsky constants, the SGS viscosity ν_{SGS} and η_{SGS} are given at each grid point of numerical computations. We consider based on our earlier studies on Hall MHD turbulence that he Smagorinsky-type model can work well for Hall MHD or extended MHD equations including the Hall term.

For the sake of calibrating the Smagorinsky constants, LESes of Hall MHD turbulence with various values of the Smagorinsky constants have been carried out with 128^3 grid points. Direct Numerical Simulations (DNSes) of Hall MHD turbulence have been also carried out by our Fourier pseudo-spectral code with the number of grid points 1024^3, for both freely decaying isotropic turbulence and forced magnetized homogeneous turbulence. In Fig.1, (a) the kinetic and (b) the magnetic energy spectra obtained by the DNS and LESes of freely decaying Hall MHD turbulence are shown. The symbols pset000, pset001, pset002, pset003 and pset004 represents that set of the Smagorinsky constants $C_\nu = C_\eta$ =0.092, 0.13, 0.046, 0.23, 0.345, respectively. It is seen that the energy specrta obtained by the LESes coincide well with the spectrum obtained by the DNS. We have carried out further calibration on the Smagorinsky, from the points of views of spatial structures of the vorticity and the current density fields in freely decaying isotropic turbulence as well as those of magnetized homogeneous Hall MHD turbulence. From these studies we consider that pset004 gives the best Smagorinsky combination in our study[1].

Here we have to note that the SGS model adopted here can be too dissipative on the parallel component of the magnetic field. The Smagorinsky constants are optimized mainly for the perpendicular components which are more excited by turbulent motions than the parallel components. In order to improve the excessive damping in the parallel direction, we are going to introducing an anisotropic effect in the Smagorinsky-type model. We are also studying more precise SGS models on the Hall term.

1) H. Miura, K.Araki, and F. Hamba: J.Comput.Phys. **316**, 385--395 (2016).

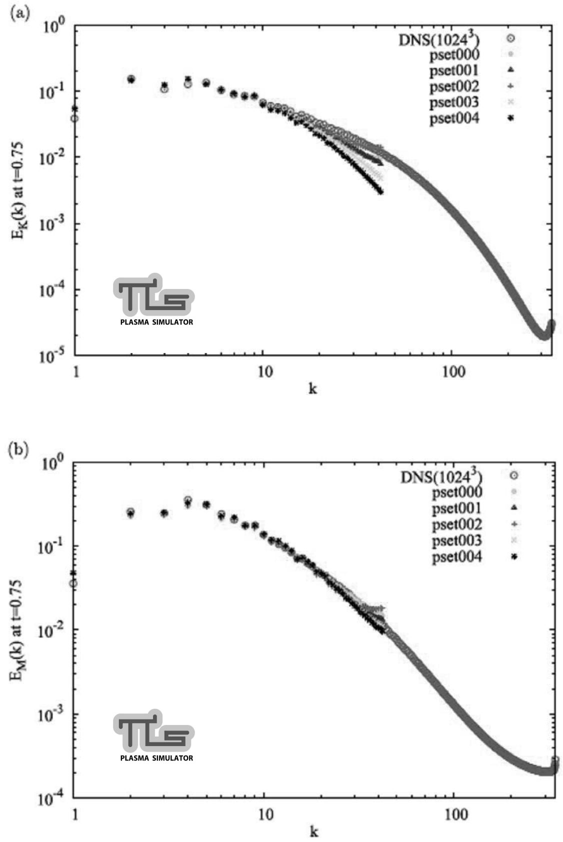

Fig. 1. (a) Kinetic and (b) magnetic energy spectra obtained by the DNS and LESes. The symbols pset000, pset001, pset002, pset003 and pset004 represents that set of the Smagorinsky constants C_ν = C_η=0.092, 0.13, 0.046, 0.23, 0.345, respectively.