§35. Time-Domain Simulation of Shielding Current Density in High-Temperature Superconductor

Kamitani, A., Saitoh, A., Takayama, T. (Yamagata Univ.),
Ikuno, S. (Tokyo Univ. of Technology),
Nakata, S. (Ritsumeikan Univ.),
Itoh, T. (Nihon Univ.),
Tamura, Y. (Konan Univ.),
Saito, S. (NIT, Kushiro College),
Fujita, Y. (Nagoya Univ.),
Nakamura, H.

Introduction Recently, a high-temperature superconducting (HTS) film has been used for numerous engineering applications: magnet, energy storage system, power cable and so on. Since evaluation of the shielding current density $\boldsymbol{j}$ is indispensable for the design of such engineering applications, several numerical methods [1, 2] have been so far developed for analyzing the shielding current density.

The purpose of the present study is to develop a fast and stable numerical method for analyzing the shielding current density in an HTS film containing cracks and to investigate its performance numerically.

Governing Equation Under the thin-plate approximation, there exists a scalar function $T(\boldsymbol{x}, t)$ such that $\boldsymbol{j} = (2/b)\nabla \times (T\boldsymbol{e}_z)$, and its time evolution is governed by the following integro-differential equation [1, 2]:

$$\mu_0 \frac{\partial}{\partial t}(\hat{W}T) + (\nabla \times \boldsymbol{E}) \cdot \boldsymbol{e}_z = -\frac{\partial}{\partial t}\langle \boldsymbol{B} \cdot \boldsymbol{e}_z \rangle. \quad (1)$$

Here, $\boldsymbol{B}/\mu_0$ and $\boldsymbol{E}$ are the applied magnetic field and the electric field, respectively, and b denotes the film thickness. In addition, $\langle\ \rangle$ denotes an average operator over the thickness and the operator $\hat{W}$ is defined by

$$\hat{W}T \equiv \frac{2T(\boldsymbol{x}, t)}{b} + \iint_{\Omega} Q(|\boldsymbol{x} - \boldsymbol{x}'|)\, T(\boldsymbol{x}', t)\, d^2\boldsymbol{x}', \quad (2)$$

where $Q(r) = -(\pi b^2)^{-1}[r^{-1} - (r^2 + b^2)^{-1/2}]$. Moreover, the power law is assumed as the J-E constitutive relation [2, 3].

Numerical Methods After discretized with time, an initial-boundary-value problem of (1) is transformed to a problem in which a nonlinear boundary-value problem is solved at each time step. Although this method can be also applied to the shielding current analysis in an HTS film containing cracks, the solution of the nonlinear problem by the Newton method is extremely time-consuming. This is mainly because a linear system with a dense matrix has to be solved at each iteration cycle of the Newton method.

Let W be an FEM matrix, corresponding to the operator $\hat{W}$, in which part of the boundary conditions is also taken into account. Apparently, W is a dense matrix. If the contribution of W is subtracted from the coefficient matrix in the linear system, the remaining matrix becomes sparse. This suggests that speedup of GMRES can be realized by executing a high-speed product of W and any n-dimensional vector $\boldsymbol{v}$. For this reason, we apply the $\mathcal{H}$-matrix method [4] to the FEM matrix W and, subsequently, the resulting approximate matrix is employed to the matrix-vector multiplication $W\boldsymbol{v}$. Typical block structures of an $\mathcal{H}$-matrix are shown in Fig. 1.

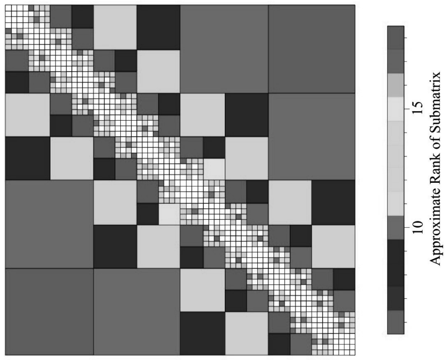

Fig. 1: Block structures of an $\mathcal{H}$-matrix. Here, full-rank and low-rank blocks are denoted by white and colored squares, respectively. In this figure, the correspondence between color level and an approximate rank of low-rank blocks is shown in the color bar.

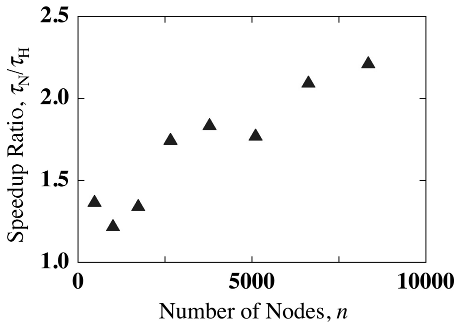

Fig. 2: Dependence of the speedup ratio τ_N/τ_H on the number of nodes n.

Let us investigate the influence of the $\mathcal{H}$-matrix method on the speedup of the shielding current analysis. To this end, the speedup ratio τ_N/τ_H is measured as functions of n and is depicted in Fig. 2. Here, τ_N and τ_H denote CPU times required for the execution of the code with and without the $\mathcal{H}$-matrix method, respectively. The speedup ratio τ_N/τ_H increases roughly with increasing number of nodes and it always exceeds unity. In other words, the $\mathcal{H}$-matrix method can accelerate the numerical code for the shielding current analysis and its speedup effect will be enhanced with an increase in the number of nodes. From these results, we can conclude that the $\mathcal{H}$-matrix method is effective for a large-sized shielding current analysis in an HTS film containing cracks.

1) Yoshida, Y. et al.: IEEE. Trans. Magn. 30 (1994) 3503.

2) Kamitani, A. et al.: IEEE Trans. Magn. 51 (2015) Art. No. 7000504.

3) Brambilla, R. et al.: IEEE Trans. Appl. Supercond. 22 (2012) Art. No. 8401006.

4) Kurz, S. et al.: IEEE Trans. Magn. 38 (2002) 412.