# §44. A Laguerre Expansion Method for the Field Particle portion in the Linearized Coulomb Collision Operator


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Recently, experimentally measured impurity flow velocities of NBI heated plasmas in Heliotron-J were successfully explained by the neoclassical transport theory [1-2]. Especially in Ref.[3], the flow determination mechanisms in multi-ion-species heliotron plasmas with the external parallel momentum input were theoretically investigated in detail. In these studies, following the standard procedure in so-called moment equation approach, the drift kinetic equations (DKE) of all thermal particle species are converted to the generalized parallel force balance equations expressed in forms of simultaneous linear algebraic equations by taking the $\left\langle \int {}_{\|} L_j^{(3/2)}(x_a^2)\mathrm{d}^3\mathbf{v} \right\rangle$ integrals of the DKEs. Here, $L_j^{(\alpha)}(K) \equiv \left(e^K K^{-\alpha}/j!\right)\mathrm{d}^j\left(e^{-K}K^{j+\alpha}\right)/\mathrm{d}K^j$ is the generalized Laguerre (Sonine) polynomial corresponding to the algebraic expression of the energy space structure, and $x_a^2 \equiv m_a v^2/\left(2\langle T_a \rangle\right)$. In this report, we explain a method used there to obtain the $\int {}_{\|} L_j^{(3/2)}(x_a^2) C_{a\mathrm{f}}\left(f_{a\mathrm{M}}, f_{\mathrm{f}}\right)\mathrm{d}^3\mathbf{v}$ integrals for the collision operators $C_{a\mathrm{f}}\left(f_{a\mathrm{M}}, f_{\mathrm{f}}\right)$ that describe collisions of thermal particles ($a$) with the fast ions' slowing down velocity distribution $f_{\mathrm{f}}(\mathbf{x},\mathbf{v})$ [4].

Since the well-known Braginskii's matrix expression of the collision is not applicable for this velocity distribution function that cannot be expressed by usual orthogonal expansion methods for the energy space, we shall start from the RMJ (Rosenbluth- MacDonald-Judd) form of the Coulomb collision operator as the most basic expression of the collision. The spherical coordinates expression of the field particle portion $C_{ab}\left(f_{a\mathrm{M}}, f_b\right)$ for the spherical harmonic expansion form of the field particles' velocity distribution

$$f_b(\mathbf{v}) = \sum_{l=0}^{\infty}\Big[a_l^0(v)P_l(\cos\theta) + \sum_{m=1}^{l} P_l^m(\cos\theta)\left\{a_l^m(v)\cos(m\phi) + b_l^m(v)\sin(m\phi)\right\}\Big] \equiv \sum_{l=0}^{\infty} f_b^{(l)}(v,\theta,\phi)$$

is given by

$$C_{ab}\left(f_{a\mathrm{M}}, f_b\right) = 4\pi\left(\frac{e_a e_b}{m_a}\right)^2 \ln\Lambda_{ab} f_{a\mathrm{M}} \frac{m_a}{T_a}\left\{\frac{4\pi T_a}{m_b} f_b - \mathcal{H}\left(f_b\right) + \left(\frac{m_a}{m_b} - 1\right) v \frac{\partial \mathcal{H}\left(f_b\right)}{\partial v} + \frac{m_a v^2}{2T_a}\frac{\partial^2 \mathcal{G}\left(f_b\right)}{\partial v^2}\right\},$$

$$\mathcal{H}(f_b) = 4\pi\sum_{l=0}^{\infty}\frac{1}{2l+1}\left\{\frac{1}{v^{l+1}}\int_0^v (v')^{l+2} f_b^{(l)}(v',\theta,\phi)\mathrm{d}v' + v^l \int_v^\infty \frac{f_b^{(l)}(v',\theta,\phi)}{(v')^{l-1}}\mathrm{d}v'\right\},$$

and

$$\frac{\partial^2 \mathcal{G}(f_b)}{\partial v^2} = 4\pi\sum_{l=0}^{\infty}\frac{1}{2l+1}\left\{\frac{(l+1)(l+2)}{2l+3}\frac{1}{v^{l+3}}\int_0^v (v')^{l+4} f_b^{(l)}(v',\theta,\phi)\mathrm{d}v' - \frac{(l-1)l}{2l-1}\frac{1}{v^{l+1}}\int_0^v (v')^{l+2} f_b^{(l)}(v',\theta,\phi)\mathrm{d}v' + \frac{(l+1)(l+2)}{2l+3} v^l \int_v^\infty \frac{f_b^{(l)}(v',\theta,\phi)}{(v')^{l-1}}\mathrm{d}v' - \frac{(l-1)l}{2l-1} v^{l-2}\int_v^\infty \frac{f_b^{(l)}(v',\theta,\phi)}{(v')^{l-3}}\mathrm{d}v'\right\}$$

General Sonine polynomial expansion coefficients of this operator $\int v^l P_l^m(\xi)\cos\left\{m(\phi-\phi_0)\right\} L_j^{(l+1/2)}(x_a^2) C_{ab}\left(f_{a\mathrm{M}}, f_b\right)\mathrm{d}^3\mathbf{v}$ with $\xi \equiv \cos\theta \equiv v_{\|}/v$ can be derived by integrations by parts for $\int_0^\infty \mathrm{d}v$ using indefinite integral formulas of $\int x^{2n+1}\exp\left(-a^2x^2\right)\mathrm{d}x$ and $\int x^{2n}\exp\left(-a^2x^2\right)\mathrm{d}x$. Although the results can reproduce also the Braginskii's matrix elements when $f_b = x_b^l P_l(\xi) L_j^{(l+1/2)}(x_b^2) f_{b\mathrm{M}}$ is substituted, in the applications to the NBI experiments, direct numerical integrals of the $\int v\xi L_j^{(3/2)}(x_a^2) C_{ab}\left(f_{a\mathrm{M}}, f_b\right)\mathrm{d}^3\mathbf{v}$ formulas were used with substituting $f_b = f_{\mathrm{f}}(\mathbf{x},\mathbf{v})$.

In this Sonine polynomial expansion procedure, we did not use an assumption of ${}_{\mathrm{i}}/m_{\mathrm{i}} \ll v_{\mathrm{b}}^2 \ll 2T_{\mathrm{e}}/m_{\mathrm{e}}$ that had been frequently used in past analytical theories on the fast ions' slowing down process. From the viewpoint of the field particle portion $C_{a\mathrm{f}}\left(f_{a\mathrm{M}}, f_{\mathrm{f}}\right)$, the assumption ${}_{\mathrm{b}}^2 \ll 2T_{\mathrm{e}}/m_{\mathrm{e}}$ previously corresponded to the use of the usual small mass ratio approximation for the electron-ion collisions also for the e-f collision $C_{\mathrm{ef}}\left(f_{\mathrm{eM}}, f_{\mathrm{f}}\right)$ in calculations of the shielding current component in the beam driven currents. For future studies requiring the $C_{a\mathrm{f}}\left(f_{a\mathrm{M}}, f_{\mathrm{f}}\right)$ of all thermal particle species $a$≠f, however, these kinds of asymptotic limit approximations giving the e-f and the i-f collision formulas separately will be confusing and inconvenient. Therefore we unified the formulas for electrons and thermal ions based on a derivation procedure allowing arbitrary energy space structures of the field particles' velocity distributions.


[1] Kobayashi,S., Lee,H.Y., et al., 25th IAEA-FEC, Saint Petersburg, Russia, 13-18 Oct. 2014, EX/P4-28

[2] Nishimura,S., Nakamura,Y., and Nishioka,K., Phys.Plasmas **22**,092505 (2015)

[3] Nishioka,K., Nakamura,Y., Nishimura,S., et al., Phys.Plasmas **23**, 032511 (2016)

[4] Nishimura, S., Phys.Plasmas **22**, 122503 (2015), ibid. **23**, 029901 (2016), NIFS-1126 (2015)